

Λύσεις Διαγωνίσματος Α' Λυκείου
30-9-2017

ΘΕΜΑ Α

- | | | | |
|-----------|-------------------------|--|---|
| <u>A1</u> | 1. $\alpha^{k+\lambda}$ | 5. 1 | 9. $(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ |
| | 2. $(\alpha\beta)^k$ | 6. -8 | 10. $(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$ |
| | 3. $\frac{1}{\alpha^k}$ | 7. $-\frac{1}{4}$ | 11. $(2\alpha-1)^2$ |
| | 4. $\alpha^{k\lambda}$ | 8. $\left(\frac{\beta}{\alpha}\right)^v$ | 12. $\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma$ |
| | | | 13. $\alpha^3-3\alpha^2\beta+3\alpha\beta^2+\beta^3$ |
| | | | 14. $4x^2-4xy^2+y^4$ |
| | | | 15. $4x^2-y^2$ |

- | | | |
|-----------|------|-------|
| <u>A2</u> | 1. Λ | 6. Σ |
| | 2. Σ | 7. Σ |
| | 3. Λ | 8. Λ |
| | 4. Λ | 9. Σ |
| | 5. Σ | 10. Λ |

ΘΕΜΑ Β

B1 1. $\alpha^2+2\alpha\beta+\beta^2 - (\alpha^2-2\alpha\beta+\beta^2) = \alpha^2+2\alpha\beta+\beta^2 - \alpha^2+2\alpha\beta-\beta^2 = 4\alpha\beta$

2. $\alpha^2 - (\alpha^2-1) = \alpha^2 - \alpha^2 + 1 = 1$

B2

1. $\frac{\alpha^3-2\alpha^2+\alpha}{\alpha^2-\alpha} = \frac{\alpha(\alpha^2-2\alpha+1)}{\alpha(\alpha-1)} = \frac{(\alpha-1)^2}{\alpha-1} = \alpha-1, \alpha \neq 0 \text{ or } \alpha \neq 1$

2. $\frac{\alpha^2-\alpha+2\alpha-2}{\alpha^2-1} = \frac{\alpha(\alpha-1)+2(\alpha-1)}{(\alpha-1)(\alpha+1)} = \frac{(\alpha-1)(\alpha+2)}{(\alpha-1)(\alpha+1)} = \frac{\alpha+2}{\alpha+1}, \alpha \neq \pm 1$

3. $\frac{\alpha^2+\alpha+1}{\alpha+1} \cdot \frac{(\alpha+1)(\alpha+1)}{(\alpha-1)(\alpha^2+2\alpha+1)} = 1, \alpha \neq \pm 1$

ΘΕΜΑ Γ

$$\underline{\underline{\Gamma_1}} \quad \pi \times 10 \text{ ΓΜ } 35 \quad \text{ΦΥΛΛΑΔΙΟ ΕΝΔ}$$

$$\underline{\underline{\Gamma_2}} \quad \pi \times 3 \text{ ΓΜ } 33 \quad \text{ΦΥΛΛΑΔΙΟ ΕΝΔ}$$

$$\underline{\underline{\Gamma_3}} \quad \pi \times 12 \text{ ΓΜ } 36 \quad \text{ΦΥΛΛΑΔΙΟ ΕΝΔ}$$

ΘΕΜΑ ΔΔ1

$$(I) \quad x = \frac{(2^{11} \cdot 2^{-12})}{2^{-6}} : (2^{18} \cdot 2^{-12}) = \frac{2^{-1}}{2^{-6}} : 2^6$$

$$= 2^{-1} \cdot 2^6 \cdot 2^{-6} = 2^{-1} = \frac{1}{2}$$

$$y = \frac{4^5 \cdot 9^6}{6^{12}} \cdot (-1) = -\frac{2^{10} \cdot 3^{12}}{2^{12} \cdot 3^{12}} = -2^{10-12} = -\frac{1}{2^2} = -\frac{1}{4}$$

$$(II) \quad A = \frac{x^{12} \cdot y^{-9} \cdot x^{-10} \cdot y^{-5}}{x^6 \cdot y^{-16}} = \frac{x^2 \cdot y^{-14}}{x^6 \cdot y^{-16}} = x^{-4} \cdot y^2 =$$

$$= \frac{y^2}{x^4} = \left(\frac{y}{x^2}\right)^2 = \left(\frac{-\frac{1}{4}}{\frac{1}{4}}\right)^2 = 1$$

Δ2 Από την ταυτότητα του Euler $a^3 + b^3 + \gamma^3 = 3ab\gamma$

$$\text{Επίσης } a+b = -\gamma \quad \text{και } (a+b)^2 = \gamma^2 \Rightarrow a^2 + 2ab + b^2 = \gamma^2$$

$$\Leftrightarrow a^2 + b^2 - \gamma^2 = -2ab$$

$$A = \frac{a^3 + b^3 + \gamma^3 - 3ab(\gamma+1)}{a^2 + b^2 - \gamma^2} = \frac{3ab\gamma - 3ab\gamma - 3ab}{-2ab}$$

$$= \frac{3\gamma - 3\gamma - 3}{-2} = \frac{-3}{-2} = \frac{3}{2}$$