ΛΥΣΕΙΣ: ΜΑΘΗΜΑΤΙΚΑ Α ΛΥΚΕΙΟΥ

3-1-2018

ΘΕΜΑ Α

Α1 / Α2 → ΘΕΩΡΙΑ

Α3. 1. Λ 2 Σ 3. Λ 4. Λ 5. Λ

ΘΕΜΑ Β

Β1 ① $(2\sqrt{2}-3\sqrt{2})(5\sqrt{2}+6\sqrt{2}-4\sqrt{2}) = (-\sqrt{2})(7\sqrt{2}) = -14$

② $5^{\frac{1}{3}} \cdot 5^{\frac{3}{2}} \cdot 5^{\frac{4}{6}} = 5^{2} \cdot 5^{\frac{1}{2}} \Leftrightarrow$

$5^{\frac{2+9+4}{6}} = 5^{\frac{5}{2}} \Leftrightarrow 5^{\frac{15}{6}} = 5^{\frac{5}{2}} \Leftrightarrow 5^{\frac{5}{2}} = 5^{5/2}$ ισχύει.

Β2 ① $2-3x=4$ ή $2-3x=-4$

$-3x = 2$ ή $-3x=-6$

$x = -2/3$ ή $x = 2$

② $x+1 = 2+3x$ ή $x+1 = -2-3x$

$-2x = 1$ ή $4x=-3$

$x = -1/2$ $x=-3/4$

③ $x^2(x^3-27)=0 \Leftrightarrow x=0$ ή $x=3$

④ Αν $x+3 \geq 0 \Leftrightarrow x \geq -3$: $x+3 = 2x-3 \Leftrightarrow -x=-6$

$\Leftrightarrow x = 6$ Δεκτή

Αν $x+3<0 \Leftrightarrow x<-3$: $-x-3 = 2x-3 \Leftrightarrow -3x=0$

$x=0$

Απορ

ΘΕΜΑ Γ

Γ1 (1) $x+1=0$ και $x^2-1=0 \Leftrightarrow x=\pm 1$

$x=-1$

άρα $\boxed{x=-1}$

(2) $|x|^2-9|x|+8=0$, $\Delta=49$ $|x|=\begin{cases} 8 \to x=\pm 8 \\ 1 \to x=\pm 1 \end{cases}$

(3) $x^4-5x^2+4=0$, $\Delta=9$ $x^2=\begin{cases} 4 \to x=\pm 2 \\ 1 \to x=\pm 1 \end{cases}$

(4) $(\lambda^2-1)x=-(\lambda-2) \Leftrightarrow (\lambda-1)(\lambda+1)x=-(\lambda-2)$

- $\lambda \neq \pm 1 \to$ ΜΟΝΑΔΙΚΗ ΛΥΣΗ $x=\dfrac{-1}{\lambda+1}$
- $\lambda=1 \to$ $0x=0$ ΑΠΕΙΡΕΣ ΛΥΣΕΙΣ
- $\lambda=-1 \to$ $0x=2$ ΑΔΥΝΑΤΗ

Γ2 (1) (α)

$$\begin{array}{r} 2 \le x \le 3 \\ 1 \le y \le 2 \quad + \\ \hline 3 \le x+y \le 5 \end{array}$$

(β) • $2 \le x \le 3 \overset{\cdot 3}{\Leftrightarrow} 6 \le 3x \le 9$

• $1 \le y \le 2 \overset{\cdot 2}{\Leftrightarrow} 2 \le 2y \le 4 \Leftrightarrow -4 \le -2y \le -2$

$6-4 \le 3x-2y \le 9-2 \Leftrightarrow 2 \le 3x-2y \le 7$

(γ) $1 \le y \le 2 \Leftrightarrow \frac{1}{2} \le \frac{1}{y} \le 1 \to \frac{2}{2} \le \frac{x}{y} \le 3 \Leftrightarrow 1 \le \frac{x}{y} \le 3$

(2) $\dfrac{9x^2-4y^2}{|2y-3x|}=\dfrac{(3x-2y)(3x+2y)}{|3x-2y|}\overset{*}{=}\dfrac{\cancel{(3x-2y)}(3x+2y)}{\cancel{3x-2y}}=3x+2y$

* Από το Γ2 (1) έχουμε $2 \le 3x-2y \le 7$ άρα $3x-2y>0$

ΘΕΜΑ Δ

Δ1 $\cdot\ |\alpha|^3 - 1 - \alpha^2 + |\alpha| \leq 0 \iff |\alpha|^3 - |\alpha|^2 + |\alpha| - 1 \leq 0$

$$\iff |\alpha|^2(|\alpha|-1) + (|\alpha|-1) \leq 0 \iff (|\alpha|-1)(|\alpha|^2+1) \leq 0$$

Εφόσον $|\alpha|^2+1 > 0$ τότε $|\alpha| \leq 1$

$\cdot\ |\beta| + 2 \leq \big||\beta| - 6\big| \iff (|\beta|+2)^2 \leq (|\beta|-6)^2 \iff$

$$\iff \cancel{|\beta|^2} + 4|\beta| + 4 \leq \cancel{|\beta|^2} - 12|\beta| + 36 \iff 16|\beta| \leq 32 \iff |\beta| \leq 2$$

Δ2. $$A = \frac{\sqrt{(\alpha+1)^2}}{\alpha+1} - \frac{\sqrt{(\beta-2)^2}}{\beta-2} = \frac{|\alpha+1|}{\alpha+1} - \frac{|\beta-2|}{\beta-2}$$

Επειδή, $|\alpha| \leq 1 \iff -1 \leq \alpha \leq 1 \rightarrow \alpha + 1 \geq 0$

$|\beta| \leq 2 \iff -2 \leq \beta \leq 2 \rightarrow \beta - 2 \leq 0$

Άρα $$A = \frac{\alpha+1}{\alpha+1} + \frac{\beta-2}{\beta-2} = 1 + 1 = 2$$

Δ3. $$\frac{x}{\alpha+1} + \frac{1}{x(\alpha+2)} = \frac{2\sqrt{}}{\alpha^2+3\alpha+2}$$

$$\iff \frac{x}{\alpha+1} + \frac{1}{x(\alpha+2)} = \frac{2\sqrt{6}}{(\alpha+1)(\alpha+2)} \overset{\cdot x(\alpha+1)(\alpha+2)}{\iff}$$

$$\iff x\cancel{(\alpha+1)}(\alpha+2)\frac{x}{\cancel{\alpha+1}} + \cancel{x}(\alpha+1)\cancel{(\alpha+2)}\frac{1}{\cancel{x(\alpha+2)}} = x\cancel{(\alpha+1)(\alpha+2)}\frac{2\sqrt{6}}{\cancel{(\alpha+1)(\alpha+2)}}$$

$$(\alpha+2)x^2 + (\alpha+1) = 2\sqrt{6}\,x \iff$$

$$(\alpha+2)x^2 + (\alpha+1) - 2\sqrt{6}\,x = 0 \iff (\alpha+2)x^2 - 2\sqrt{6}\,x + (\alpha+1) = 0$$

Αρκεί ν.δ.ο. $\Delta \geq 0 \iff 4\cdot 6 - 4(\alpha+2)(\alpha+1) \geq 0 \iff$

$$\iff 6 - (\alpha+2)(\alpha+1) \geq 0 \iff (\alpha+2)(\alpha+1) \leq 6 \ \overset{*}{\underline{\text{ισχύει}}}$$

*

$$-1 \leq \alpha \leq 1 \overset{+1}{\iff} 0 \leq \alpha + 1 \leq 2$$

$$-1 \leq \alpha \leq 1 \overset{+2}{\iff} 1 \leq \alpha + 2 \leq 3 \quad (\times)$$

$$0 \leq (\alpha+1)(\alpha+2) \leq 6$$

II). • έστω $\sqrt[6]{10} > \sqrt[3]{3} \iff \left(\sqrt[6]{10}\right)^6 > \left(\sqrt[3]{3}\right)^6$

$\iff 10 > 3^2$ ισχύει

• έστω $\sqrt[3]{3} > \sqrt{2} \iff \left(\sqrt[3]{3}\right)^6 > \left(\sqrt{2}\right)^6 \iff 9 > 8$ ισχύει

άρα $\sqrt[6]{10} > \sqrt[3]{3} > \sqrt{2}$

<u>Δ4</u> • $\sqrt{31+10\sqrt{6}} = \sqrt{31 + 2\cdot 5\sqrt{6}} = \sqrt{25+6+2\cdot 5\sqrt{6}}$

$= \sqrt{5^2 + (\sqrt{6})^2 + 2\cdot 5\cdot\sqrt{6}} = \sqrt{(5+\sqrt{6})^2} = 5+\sqrt{6}$

• $\dfrac{2\sqrt{2}-3\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \dfrac{(2\sqrt{2}-3\sqrt{3})(\sqrt{2}+\sqrt{3})}{2-3} =$

$= \dfrac{2\cdot 2 + 2\sqrt{2}\sqrt{3} - 3\sqrt{3}\sqrt{2} - 3\cdot 3}{-1} = \dfrac{4-9-\sqrt{6}}{-1} = \dfrac{-5-\sqrt{6}}{-1} = 5+\sqrt{6}$

Όποτε $d(\beta^5, -32) + \sqrt[2018]{2-|\beta|} = 0 \iff$

$d(\beta^5,-32) = 0$ και $2 - |\beta| = 0$

$|\beta^5 + 32| = 0$ και $|\beta| = 2$

$\beta^5 + 32 = 0$ και $\beta = 2$ ή -2

$\beta^5 = -32$

$\beta = -2$

άρα $\beta = -2$