ΛΥΣΗΣ Α ΛΥΚΕΙΟΥ ΜΑΘΗΜΑΤΙΚΑ

29/9/2018

ΘΕΜΑ 1

Α

1. Λ
2. Σ
3. Λ
4. Λ
5. Σ
6. Λ
7. Λ
8. Σ
9. Λ
10. Λ

Β

- $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$
- $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$
- $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
- $\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$
- $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma$

ΘΕΜΑ 2

Α ΛΥΜΕΝΟ ΠΧ 1 ΣΕΛ. 31

Β (1.)

$$\frac{\alpha^3-2\alpha^2+\alpha}{\alpha^2-\alpha} = \frac{\alpha(\alpha^2-2\alpha+1)}{\alpha(\alpha-1)} \overset{\substack{\alpha\neq 0\\ \alpha\neq 1}}{=\!=} \frac{\alpha(\alpha-1)^2}{\alpha(\alpha-1)} = \alpha-1$$

(2)

$$\frac{\alpha^2-\alpha+2\alpha-2}{\alpha^2-1} = \frac{\alpha(\alpha-1)+2(\alpha-1)}{(\alpha-1)(\alpha+1)} \overset{\substack{\alpha\neq 1\\ \alpha\neq -1}}{=\!=} \frac{(\alpha-1)(\alpha+2)}{(\alpha-1)(\alpha+1)} = \frac{\alpha+2}{\alpha+1}$$

(3)

$$\frac{\alpha^2+\alpha+1}{\alpha+1}\cdot\frac{(\alpha-1)(\alpha+1)}{(\alpha-1)(\alpha^2+\alpha+1)} \overset{\alpha\neq\pm 1}{=\!=} 1$$

ΘΕΜΑ 3

Α ΛΥΜΕΝΟ ΠΧ 12 ΣΕΛ. 36

Β ΛΥΜΕΝΟ ΠΧ 10 ΣΕΛ. 35

ΘΕΜΑ 4

① $A = \left[\alpha^{-4} \cdot \beta^{-6} \cdot \alpha^{4} \cdot \beta^{12} \right] : \left(\frac{\beta^{-3}}{\alpha^{9}} \right)$

$= \alpha^{-4} \cdot \alpha^{4} \cdot \beta^{-6} \cdot \beta^{12} \cdot \frac{\alpha^{9}}{\beta^{-3}}$

$= \alpha^{9} \cdot \beta^{6} \cdot \beta^{3} = \alpha^{9} \cdot \beta^{9} = (\alpha \cdot \beta)^{9}$

② $A = (\alpha \cdot \beta)^{9} = \left(2018 \cdot \frac{1}{2018} \right)^{9} = 1^{9} = 1$

③ Γιατί: $\left(y + \frac{1}{y^2} \right)^2 = 3 \Leftrightarrow y^2 + 2y \frac{1}{y} + \frac{1}{y^2} = 3$

$\Leftrightarrow y^2 + 2 + \frac{1}{y^2} = 3 \Leftrightarrow y^2 + \frac{1}{y^2} = 1 = B$

• $\Gamma = y^3 + \frac{1}{y^3} = \left(y + \frac{1}{y} \right) \left(y^2 - y \cdot \frac{1}{y} + \frac{1}{y^2} \right) = \left(y + \frac{1}{y} \right) (1 - 1) = 0$

④ ΠΑΡΑΤΗΡΟΥΜΕ ΟΤΙ:

$(2x + 5) + (2x - 7) + (2 - 4x) =$

$2x + 5 + 2x + 2 - 4x - 7 = 4x - 4x + 7 - 7 = 0$

ΟΠΟΤΕ ΑΠΟ ΤΗΝ ΤΑΥΤΟΤΗΤΑ ΤΟΥ EYLER

ΕΠΟΜΕΝΩΣ: $(2x + 5)^3 + (2x - 7)^3 + (2 - 4x)^3 = 3(2x + 5)(2x - 7)(2 - 4x)$

ΟΠΟΤΕ: $\Delta = \frac{3x + 1}{3(2x + 5)(2x - 7)(2 - 4x)}$

ΑΡΑ $\begin{cases} x \neq -5/2 \\ x \neq 7/2 \\ x \neq 1/2 \end{cases}$