

ΛΥΣΕΙΣ ΤΡΟΣΟΜΟΙ ΩΣΗΣ Β' ΤΑΞΗΣ ΛΥΚΕΙΟΥ

ΘΕΜΑ Α

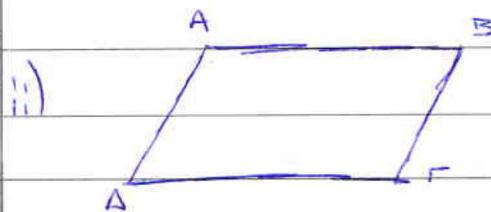
A<sub>1</sub>

i) a)  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos(\vec{a}, \vec{b}) = \sqrt{12} \cdot 3 \cdot \frac{\sqrt{2}}{2} = 6$

b)  $\vec{a}^2 \cdot \vec{b}^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 = (2\sqrt{3})^2 \cdot 3^2 = 8 \cdot 9 = 72$

c)  $(3\vec{a}) \cdot \vec{b} = 3\vec{a} \cdot \vec{b} = 3 \cdot 6 = 18$

d)  $(\vec{a} \cdot 2\vec{b})^2 = (2 \cdot \vec{a} \cdot \vec{b})^2 = (2 \cdot 6)^2 = 144.$



$$\vec{AF} = \vec{AB} + \vec{BF} = 5\vec{a} + 2\vec{b} + \vec{a} - 3\vec{b} = 6\vec{a} - \vec{b}$$

$$\vec{AB} = \vec{AF} - \vec{BF} = 5\vec{a} + 2\vec{b} - (\vec{a} - 3\vec{b}) = 4\vec{a} + 5\vec{b}$$

iii)  $|\vec{AF}|^2 = |6\vec{a} - \vec{b}|^2 = (6\vec{a} - \vec{b})^2 = 36\vec{a}^2 - 12\vec{a} \cdot \vec{b} + \vec{b}^2 =$   
 $= 36 \cdot 8 - 12 \cdot 6 + 9 = 6 \cdot 36 + 9 = 25 \cdot 9$

$$\Rightarrow |\vec{AF}| = 15$$

A<sub>2</sub>

i)  $\vec{a} \cdot \vec{b} = (3, -2) \cdot (6, 9) = 18 - 18 = 0 \quad \text{dpa} \quad \vec{a} \perp \vec{b} \Rightarrow \text{ΛΑΟΣ.}$

ii)  $\vec{a} \perp \vec{b} \text{ orice } \vec{a} \cdot \vec{b} = 0 \Leftrightarrow (k^2+5, -2) \cdot (1, k+2) = 0$

$$\Leftrightarrow k^2 + 5 - 2k - 4 = 0 \Leftrightarrow k^2 - 2k + 1 = 0$$

$$\Leftrightarrow (k-1)^2 = 0 \Leftrightarrow k=1 \Rightarrow \text{ΖΩΤΟ.}$$

Παρατηρήσεις

ΘΕΜΑ Β

B<sub>1</sub>

i) ΕΓΓΩ<sub>καν</sub>  $\begin{cases} \lambda^2 + \lambda - 2 = 0 \Rightarrow \lambda = 1 \text{ ή } \lambda = -2 \\ \lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda = 1 \text{ ή } \lambda = 2 \end{cases} \Rightarrow \lambda = 1$

οπός για να παριστάνεται ειδεία πρέπει  $\lambda \neq 1$ .

ii) Αφού  $E/\gamma/\gamma$  έχει λύρα  $E = x = x_0$  οπός πρέπει  
ο διεισδετής του γ να είναι λιγότερος.

$$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \begin{cases} \lambda = 1 \text{ Άπορ} \\ \lambda = 2 \end{cases}$$

iii) Το  $(0,0)$  επομήθευει την εξισωση.

(1)  $\frac{x=0}{\omega=0} \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \begin{cases} \lambda = 1 \text{ Άπορ} \\ \lambda = -1 \end{cases}$

iv) ΕΓΓΩ  $\vec{\delta}_1 \parallel \varepsilon_1$  ότι  $\vec{\delta}_1 = (-B, A) = (-\lambda^2 + 3\lambda - 2, \lambda^2 + \lambda - 2)$

ΕΓΓΩ  $\vec{\delta}_2 \parallel \varepsilon_2$  ότι  $\vec{\delta}_2 = (-B, A) = (1, 1)$

$$E \parallel \varepsilon_2 \Leftrightarrow \vec{\delta}_1 \parallel \vec{\delta}_2 \Leftrightarrow \det(\vec{\delta}_1, \vec{\delta}_2) = 0 \Leftrightarrow \begin{vmatrix} -\lambda^2 + 3\lambda - 2 & \lambda^2 + \lambda - 2 \\ 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -\lambda^2 + 3\lambda - 2 - \lambda^2 - \lambda + 2 = 0$$

$$\Leftrightarrow -2\lambda^2 + 2\lambda = 0 \Leftrightarrow 2\lambda(-\lambda + 1) = 0 \Leftrightarrow \begin{cases} \lambda = 0 \\ \lambda = 1 \text{ Άπορ} \end{cases}$$

B2

a) Το σημείο τοπού των  $\varepsilon_1, \varepsilon_2$ :

$$\begin{cases} \varepsilon_1: 2x - 5y + 3 = 0 \\ \varepsilon_2: x - 3y - 7 = 0 \end{cases} \xrightarrow{\text{(-2)}} \begin{cases} 2x - 5y + 3 = 0 \\ -2x + 6y + 14 = 0 \end{cases} \xrightarrow{(+)} \begin{cases} 2x - 5y + 3 = 0 \\ 6y + 14 = 0 \end{cases}$$

$$6y + 14 = 0 \Rightarrow y = -\frac{14}{6} \xrightarrow{\text{ε2}} x - 3(-\frac{14}{6}) - 7 = 0$$

$$\Rightarrow x = -44$$

Το σημείο τοπού  $A(-44, -\frac{14}{6})$

$$\varepsilon \perp \varepsilon_3 \Leftrightarrow \lambda_{\varepsilon} \cdot \lambda_{\varepsilon_3} = -1 \Leftrightarrow \lambda_{\varepsilon} \cdot (-4) = -1 \Leftrightarrow \lambda_{\varepsilon} = \frac{1}{4}.$$

$$\varepsilon: y - (-\frac{14}{6}) = \frac{1}{4}(x - (-44)) \Leftrightarrow y + \frac{14}{6} = \frac{1}{4}(x + 44)$$

$$\Leftrightarrow 4y + 68 = x + 44 \Leftrightarrow \boxed{x - 4y - 24 = 0}$$

$$b) \lambda = \varepsilon \varphi \omega = \varepsilon \varphi \frac{3\pi}{4} = \varepsilon \varphi (\pi - \frac{\pi}{4}) = -\varepsilon \varphi \frac{\pi}{4} = -1$$

$$\varepsilon: y - (-1) = -1(x - 1) \Leftrightarrow \boxed{y = -x}$$

c) Το μέσο του  $AB: M(\frac{2+4}{2}, \frac{3+5}{2}) = M(3, 4)$

$$\varepsilon \perp AB \Leftrightarrow \lambda_{\varepsilon} \cdot \lambda_{AB} = -1 \Leftrightarrow \lambda_{\varepsilon} \cdot \frac{y_B - y_A}{x_B - x_A} = -1$$

$$\Leftrightarrow \lambda_{\varepsilon} \cdot \frac{5 - 3}{4 - 2} = -1 \Leftrightarrow \boxed{\lambda_{\varepsilon} = -1}$$

$$\varepsilon: y - 4 = -1 \cdot (x - 3) \Leftrightarrow \boxed{y = -x + 7}$$

Omega  $\Gamma$

$$\Gamma_1 \quad (\Sigma_1) : \left\{ \begin{array}{l} x = -1 - y \\ xy = -6 \end{array} \right. \Leftrightarrow (-1-y) \cdot y = -6 \Leftrightarrow (1+y)y = 6$$

$$\Leftrightarrow y + y^2 = 6 \Leftrightarrow y^2 + y - 6 = 0, \Delta = 25 \quad y = \begin{cases} 2 \rightarrow (x, y) = (-3, 2) \\ -3 \rightarrow (x, y) = (2, -3) \end{cases}$$

$$\Gamma_2 \quad a) \quad 4w^2x - 1 = 0 \quad | \quad 26wx + 1 = 0 \quad | \quad 6qx = 0$$

$$w^2x = \frac{1}{4} \quad | \quad 6wx = -\frac{1}{2} \quad | \quad 6qx = 6q \cdot \frac{1}{2}$$

$$w^2x = \pm \frac{1}{2} \quad | \quad 6wx = 6w(n - \frac{n}{3}) \quad | \quad x = kn + \frac{n}{2} \quad k \in \mathbb{Z}$$

$$w^2x = w \frac{n}{6} \quad | \quad w^2x = w(-\frac{n}{6})$$

$$x = 2kn + \frac{n}{6} \quad | \quad x = 2kn - \frac{n}{6} \quad k \in \mathbb{Z}$$

$$x = 2kn + \frac{5n}{6} \quad | \quad x = 2kn + \frac{7n}{6}$$

$$b) \quad 2w^2x - 36wx - 3 = 0 \Leftrightarrow 2 - 26wx - 36wx - 3 = 0$$

$$\Leftrightarrow -26wx - 36wx - 1 = 0$$

$$\Delta = 9 - 8 = 1 \quad 6wx = \frac{3+1}{-4} = \frac{4}{-4} = -1 \rightarrow x = 2kn \pm n \quad k \in \mathbb{Z}$$

$$\frac{3-1}{-4} = \frac{2}{-4} = -\frac{1}{2} \rightarrow x = 2kn \pm \frac{2n}{3}$$

$$\Gamma_3 \quad \cdot w \frac{5n}{4} = w(n + \frac{n}{4}) = w \frac{n}{4} = -\frac{\sqrt{2}}{2}$$

$$w^2 \frac{n}{3} = w(n - \frac{n}{3}) = w \frac{n}{3} = \frac{\sqrt{3}}{2}$$

$$\cdot 6w \frac{7n}{6} = 6w(n + \frac{n}{6}) = -6w \frac{n}{6} = -\frac{\sqrt{3}}{2}$$

$$6q \frac{3n}{4} = 6q(n - \frac{n}{4}) = -6q \frac{n}{4} = -\frac{\sqrt{3}}{2}$$

$$\cdot 6q \frac{4n}{3} = 6q(n + \frac{n}{3}) = 6q \frac{n}{3} = \frac{\sqrt{3}}{2}$$

$$6q \frac{5n}{6} = 6q(n - \frac{n}{6}) = -6q \frac{n}{6} = -\frac{\sqrt{3}}{2}$$

$$\cdot 6w \frac{3n}{4} = 6w(n - \frac{n}{4}) = -6w \frac{n}{4} = -\frac{\sqrt{2}}{2}$$

$$w(-\frac{n}{6}) = -w \frac{n}{6} = -\frac{1}{2}$$

$$A = \frac{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{1 \cdot \frac{1}{2}} = \frac{\frac{2}{4}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$$

•  $\delta_{\text{MAX}} \Delta$

$$\begin{aligned} \Delta L & \left(1 + \frac{1}{6\varphi w}\right)^2 + \left(1 - \frac{1}{6\varphi w}\right)^2 = (1 + \varepsilon \varphi w)^2 + (1 - \varepsilon \varphi w)^2 = \\ & = 1 + 2\varepsilon \varphi w + \varepsilon^2 \varphi^2 w^2 + 1 - 2\varepsilon \varphi w + \varepsilon^2 \varphi^2 w^2 = 2(1 + \varepsilon^2 \varphi^2 w^2) \\ & = \frac{2}{6w^2} \end{aligned}$$

$$\begin{aligned} \Delta 2 \quad u(sn+w) &= u(n+w) = -u\varphi w \quad \left| \begin{array}{l} 6\varphi(sn+w) = 6\varphi(n+w) = 6\varphi w \\ 6w(sn-w) = 6w(n-w) = -6vww \end{array} \right. \\ u\varphi\left(\frac{sn}{2}-w\right) &= u\varphi\left(\frac{n}{2}-w\right) = 6ww \quad \left| \begin{array}{l} u\varphi(n-w) = u\varphi w \\ 6w\left(\frac{n}{2}-w\right) = u\varphi w \end{array} \right. \\ 6w\left(\frac{2n}{2}+w\right) &= 6w\left(\frac{3n}{2}+w\right) = u\varphi w \quad \left| \begin{array}{l} 6\varphi\left(\frac{3n}{2}+w\right) = -\varepsilon\varphi w \end{array} \right. \end{aligned}$$

$$\bullet \quad \frac{-6vw^2w}{\varepsilon\varphi w \ 6\varphi w} = -6w^2w = u\varphi^2w - 1$$

$$\Delta 3 \quad \text{UPRIM: } x - \frac{\pi}{4} \neq kn + \frac{\pi}{2} \Rightarrow x \neq kn + \frac{\pi}{2} + \frac{\pi}{4} \Rightarrow \boxed{x \neq kn + \frac{3\pi}{4}}$$

$$x + \frac{5\pi}{3} \neq kn \Rightarrow \boxed{x \neq kn - \frac{5\pi}{3}}$$

$$\alpha) \quad 6\varphi\left(\frac{\pi}{2} - x + \frac{\pi}{4}\right) = 6\varphi\left(x + \frac{5\pi}{3}\right) \Leftrightarrow 6\varphi\left(\frac{3\pi}{4} - x\right) = 6\varphi\left(x + \frac{5\pi}{3}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{3\pi}{4} - x = kn + x + \frac{5\pi}{3} \Leftrightarrow -2x = kn + \frac{5\pi}{3} - \frac{3\pi}{4} \Leftrightarrow -2x = kn + \frac{10n - 9n}{12}$$

$$\Leftrightarrow x = -\frac{kn}{2} - \frac{11n}{24} \quad k \in \mathbb{Z}$$

$$\beta) \quad 6w\left(2x + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow 6w\left(2x + \frac{\pi}{6}\right) = 6w\left(\frac{5\pi}{6}\right) \Leftrightarrow 2x = 2kn \pm \frac{5\pi}{6} - \frac{\pi}{6}, \quad k \in \mathbb{Z}$$

$$\boxed{x = kn + \frac{\pi}{3}}$$

$$-\pi < x < \pi$$

$$\vdots$$

$$-\frac{\pi}{3} < k < \frac{2\pi}{3}$$

$$\boxed{x = kn - \frac{\pi}{2}}$$

$$-\pi < x < \pi$$

$$\vdots$$

$$-\frac{1}{2} < k < \frac{3}{2}$$

$$\alpha \alpha \quad x = -\frac{2\pi}{3} \quad x = \pi/3$$

$$x = -\frac{\pi}{2} \quad x = \frac{\pi}{2}$$

$$\Delta 4 \quad 6\varphi \frac{8\pi}{7} \cdot 6\varphi \frac{\pi}{14} + u\varphi^2 \frac{\pi}{7} + u\varphi^2 \frac{5\pi}{14} =$$

$$= \varepsilon\varphi \frac{\pi}{14} \cdot 6\varphi \frac{\pi}{14} + u\varphi^2 \frac{\pi}{7} + 6w^2 \frac{\pi}{7} = 1 + 1 = 2$$