Παρατηρήσεις

ΛΥΣΕΙΣ ΠΡΟΣΟΜΟΙΩΣΗΣ Β΄ ΤΑΞΗΣ ΛΥΚΕΙΟΥ

ΘΕΜΑ Α

Α1

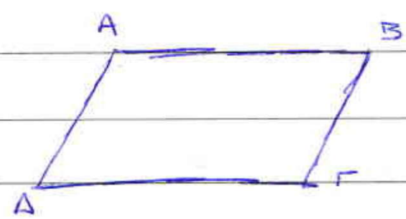
i) α) $\vec{\alpha}\cdot\vec{\beta} = |\vec{\alpha}||\vec{\beta}|\sigma\upsilon\nu(\widehat{\vec{\alpha},\vec{\beta}}) = \cancel{2}\sqrt{2}\cdot 3\cdot\frac{\sqrt{2}}{\cancel{2}} = 6$

β) $\vec{\alpha}^2\cdot\vec{\beta}^2 = |\vec{\alpha}|^2\cdot|\vec{\beta}|^2 = (2\sqrt{2})^2\cdot 3^2 = 8\cdot 9 = 72$

γ) $(3\vec{\alpha})\cdot\vec{\beta} = 3\vec{\alpha}\vec{\beta} = 3\cdot 6 = 18$

δ) $(\vec{\alpha}\cdot 2\vec{\beta})^2 = (2\cdot\vec{\alpha}\vec{\beta})^2 = (2\cdot 6)^2 = 144.$

ii)



$\vec{A\Gamma} = \vec{AB} + \vec{A\Delta} = 5\vec{\alpha}+2\vec{\beta} + \vec{\alpha} - 3\vec{\beta} = 6\vec{\alpha} - \vec{\beta}$

$\vec{\Delta B} = \vec{AB} - \vec{A\Delta} = 5\vec{\alpha}+2\vec{\beta} - (\vec{\alpha}-3\vec{\beta}) = 4\vec{\alpha}+5\vec{\beta}$

iii) $|\vec{A\Gamma}|^2 = |6\vec{\alpha}-\vec{\beta}|^2 = (6\vec{\alpha}-\vec{\beta})^2 = 36\vec{\alpha}^2 - 12\vec{\alpha}\vec{\beta} + \vec{\beta}^2 =$

$= 36\cdot 8 - 12\cdot 6 + 9 = 6\cdot 36 + 9 = 25\cdot 9$

$\Leftrightarrow |\vec{A\Gamma}| = 15$

Α2

i) $\vec{\alpha}\cdot\vec{\beta} = (3,-2)\cdot(6,9) = 18-18 = 0$ άρα $\vec{\alpha}\perp\vec{\beta}$ $\Rightarrow$ ΛΑΘΟΣ

ii) $\vec{\alpha}\perp\vec{\beta}$ οπότε $\vec{\alpha}\vec{\beta} = 0 \Leftrightarrow (\kappa^2+5, -2)(1, \kappa+2) = 0$

$\Leftrightarrow \kappa^2+5-2\kappa-4 = 0 \Leftrightarrow \kappa^2-2\kappa+1 = 0$

$\Leftrightarrow (\kappa-1)^2 = 0 \Leftrightarrow \kappa = 1$ $\Rightarrow$ ΣΩΣΤΟ.

Παρατηρήσεις

ΘΕΜΑ Β

Β1

i) Έστω $\begin{cases} \lambda^2+\lambda-2=0 \Leftrightarrow \lambda=1 \text{ ή } \lambda=-2 \\ \text{και} \\ \lambda^2-3\lambda+2=0 \Leftrightarrow \lambda=1 \text{ ή } \lambda=2 \end{cases} \Bigg\rangle \lambda=1$

οπότε για να παριστάνει ευθεία πρέπει $\lambda \neq 1$

ii) Αφού $\varepsilon // y'y$ έχει μορφή $\varepsilon: x=x_0$ οπότε πρέπει ο συντελεστής του y να είναι μηδέν.

$\lambda^2-3\lambda+2=0 \Leftrightarrow \begin{cases} \lambda=1 \text{ Απορ} \\ \text{ή} \\ \boxed{\lambda=2} \end{cases}$

iii) Το $(0,0)$ επαληθεύει την εξίσωση.

$(1) \xrightarrow[y=0]{x=0} \lambda^2-1=0 \Leftrightarrow \begin{cases} \lambda=1 \text{ Απορ} \\ \text{ή} \\ \boxed{\lambda=-1} \end{cases}$

iv) έστω $\vec{\delta_1} // \varepsilon_1$ με $\vec{\delta_1}=(-B,A)=(-\lambda^2+3\lambda-2, \lambda^2+\lambda-2)$

έστω $\vec{\delta_2} // \varepsilon_2$ με $\vec{\delta_2}=(-B,A)=(1,1)$

$\varepsilon_1 // \varepsilon_2 \Leftrightarrow \vec{\delta_1} // \vec{\delta_2} \Leftrightarrow \det(\vec{\delta_1},\vec{\delta_2})=0 \Leftrightarrow \begin{vmatrix} -\lambda^2+3\lambda-2 & \lambda^2+\lambda-2 \\ 1 & 1 \end{vmatrix}=0$

$\Leftrightarrow -\lambda^2+3\lambda-\cancel{2}-\lambda^2-\lambda+\cancel{2}=0$

$\Leftrightarrow -2\lambda^2+2\lambda=0 \Leftrightarrow 2\lambda(-\lambda+1)=0 \Leftrightarrow \begin{cases} \boxed{\lambda=0} \\ \text{ή} \\ \lambda=1 \text{ Απορ} \end{cases}$

Εν Δυνάμει
ΦΡΟΝΤΙΣΤΗΡΙΑ

Παρατηρήσεις

B2

α) Το σημείο τομής των $\varepsilon_1, \varepsilon_2$:

$$\left.\begin{cases} \varepsilon_1: 2x-5y+3=0 \\ \varepsilon_2: x-3y-7=0 \overset{\cdot(-2)}{\Longleftrightarrow} -2x+6y+14=0 \end{cases}\right\} \overset{(+)}{\Longrightarrow}$$

$$y+17=0 \Leftrightarrow y=-17 \overset{\varepsilon_2}{\Longrightarrow} x-3(-17)-7=0$$

$$\Leftrightarrow x=-44$$

Το σημείο τομής $A(-44,-17)$

$$\varepsilon \perp \varepsilon_3 \Leftrightarrow \lambda_\varepsilon \cdot \lambda_{\varepsilon_3}=-1 \Leftrightarrow \lambda_\varepsilon \cdot (-4)=-1 \Leftrightarrow \lambda_\varepsilon = 1/4.$$

$$\varepsilon: y-(-17)=\frac{1}{4}(x-(-44)) \Leftrightarrow y+17=\frac{1}{4}(x+44)$$

$$\Leftrightarrow 4y+68=x+44 \Leftrightarrow \boxed{x-4y-24=0}$$

β) $\lambda = \varepsilon\varphi\omega = \varepsilon\varphi\frac{3\pi}{4} = \varepsilon\varphi(\pi - \pi/4) = -\varepsilon\varphi\pi/4 = -1$

$$\varepsilon: y-(-1)=-1(x-1) \Leftrightarrow \boxed{y=-x}$$

γ) Το μέσο του AB: $M\left(\frac{2+4}{2}, \frac{3+5}{2}\right) \Rightarrow M(3,4)$

$$\varepsilon \perp AB \Leftrightarrow \lambda_\varepsilon \cdot \lambda_{AB}=-1 \Leftrightarrow \lambda_\varepsilon \cdot \frac{y_B-y_A}{x_B-x_A}=-1$$

$$\Leftrightarrow \lambda_\varepsilon \cdot \frac{5-3}{4-2}=-1 \Leftrightarrow \boxed{\lambda_\varepsilon=-1}$$

$$\varepsilon: y-4=-1\cdot(x-3) \Leftrightarrow \boxed{y=-x+7}$$

ΘΕΜΑ Γ

Γ1 $(\Sigma_1): \begin{cases} x = -1-y \\ xy = -6 \end{cases} \Leftrightarrow (-1-y)\cdot y = -6 \Leftrightarrow (1+y)y = 6$

$\Leftrightarrow y + y^2 = 6 \Leftrightarrow y^2 + y - 6 = 0, \quad \Delta = 25 \quad y = \begin{cases} 2 \rightarrow (x,y) = (-3,2) \\ -3 \rightarrow (x,y) = (2,-3) \end{cases}$

Γ2 α) $4\eta\mu^2 x - 1 = 0$ ή $2\sigma\upsilon\nu x + 1 = 0$ ή $\sigma\varphi x = 0$

$\eta\mu^2 x = \frac{1}{4}$ ή $\sigma\upsilon\nu x = -\frac{1}{2}$ ή $\sigma\varphi x = \sigma\varphi \frac{\pi}{2}$

$\eta\mu x = \pm \frac{1}{2}$

$\sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{3}\right)$

$\sigma\upsilon\nu x = \sigma\upsilon\nu \frac{2\pi}{3}$

$x = 2k\pi \pm \frac{2\pi}{3}$

$x = k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z}$

$\eta\mu x = \eta\mu \frac{\pi}{6}$ ή $\eta\mu x = \eta\mu\left(-\frac{\pi}{6}\right)$

$x = 2k\pi + \frac{\pi}{6}$ ή $x = 2k\pi - \frac{\pi}{6}$ $\quad k \in \mathbb{Z}$

$x = 2k\pi + \frac{5\pi}{6}$ ή $x = 2k\pi + \frac{7\pi}{6}$

β) $2\eta\mu^2 x - 3\sigma\upsilon\nu x - 3 = 0 \Leftrightarrow 2 - 2\sigma\upsilon\nu^2 x - 3\sigma\upsilon\nu x - 3 = 0$

$\Leftrightarrow -2\sigma\upsilon\nu^2 x - 3\sigma\upsilon\nu x - 1 = 0$

$\Delta = 9 - 8 = 1 \quad \sigma\upsilon\nu x = \begin{cases} \frac{3+1}{-4} = \frac{4}{-4} = -1 \rightarrow x = 2k\pi \pm \pi \\ \frac{3-1}{-4} = \frac{2}{-4} = -\frac{1}{2} \rightarrow x = 2k\pi \pm \frac{2\pi}{3} \end{cases} \quad k \in \mathbb{Z}$

Γ3

- $\eta\mu \frac{5\pi}{4} = \eta\mu\left(\pi + \frac{\pi}{4}\right) = -\eta\mu \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$
- $\sigma\upsilon\nu \frac{7\pi}{6} = \sigma\upsilon\nu\left(\pi + \frac{\pi}{6}\right) = -\sigma\upsilon\nu \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$
- $\varepsilon\varphi \frac{4\pi}{3} = \varepsilon\varphi\left(\pi + \frac{\pi}{3}\right) = \varepsilon\varphi \frac{\pi}{3} = \sqrt{3}$
- $\sigma\upsilon\nu \frac{3\pi}{4} = \sigma\upsilon\nu\left(\pi - \frac{\pi}{4}\right) = -\sigma\upsilon\nu \frac{\pi}{4} = -\sqrt{2}/2$

$\eta\mu \frac{2\pi}{3} = \eta\mu\left(\pi - \frac{\pi}{3}\right) = \eta\mu \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\varepsilon\varphi \frac{3\pi}{4} = \varepsilon\varphi\left(\pi - \frac{\pi}{4}\right) = -\varepsilon\varphi \frac{\pi}{4} = -1$

$\sigma\varphi \frac{5\pi}{6} = \sigma\varphi\left(\pi - \frac{\pi}{6}\right) = -\sigma\varphi \frac{\pi}{6} = -\sqrt{3}$

$\eta\mu\left(-\frac{\pi}{6}\right) = -\eta\mu \frac{\pi}{6} = -\frac{1}{2}$

$$A = \frac{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}{1\cdot\frac{1}{2}} = \frac{\frac{2}{4}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$$

ΘΕΜΑ Δ

Δ1

$$\left(1+\frac{1}{\sigma\varphi\omega}\right)^2+\left(1-\frac{1}{\sigma\varphi\omega}\right)^2=(1+\varepsilon\varphi\omega)^2+(1-\varepsilon\varphi\omega)^2=$$

$$=1+\cancel{2\varepsilon\varphi\omega}+\varepsilon\varphi^2\omega+1-\cancel{2\varepsilon\varphi\omega}+\varepsilon\varphi^2\omega=2(1+\varepsilon\varphi^2\omega)$$

$$=\frac{2}{\sigma\upsilon\nu^2\omega}$$

Δ2

$\eta\mu(5\pi+\omega)=\eta\mu(\pi+\omega)=-\eta\mu\omega$

$\sigma\upsilon\nu(7\pi-\omega)=\sigma\upsilon\nu(\pi-\omega)=-\sigma\upsilon\nu\omega$

$\eta\mu\left(\frac{5\pi}{2}-\omega\right)=\eta\mu\left(\frac{\pi}{2}-\omega\right)=\sigma\upsilon\nu\omega$

$\sigma\upsilon\nu\left(\frac{7\pi}{2}+\omega\right)=\sigma\upsilon\nu\left(\frac{3\pi}{2}+\omega\right)=\eta\mu\omega$

$\sigma\varphi(5\pi+\omega)=\sigma\varphi(\pi+\omega)=\sigma\varphi\omega$

$\eta\mu(\pi-\omega)=\eta\mu\omega$

$\sigma\upsilon\nu\left(\frac{\pi}{2}-\omega\right)=\eta\mu\omega$

$\sigma\varphi\left(\frac{3\pi}{2}+\omega\right)=-\varepsilon\varphi\omega$

- $$\frac{-\sigma\upsilon\nu^2\omega}{\varepsilon\varphi\omega\,\sigma\varphi\omega}=-\sigma\upsilon\nu^2\omega=\eta\mu^2\omega-1$$

Δ3 Περιορ.: $x-\frac{\pi}{4}\neq\kappa\pi+\frac{\pi}{2}\Rightarrow x\neq\kappa\pi+\frac{\pi}{2}+\frac{\pi}{4}\Rightarrow\boxed{x\neq\kappa\pi+\frac{3\pi}{4}}$

$x+\frac{5\pi}{3}\neq\kappa\pi\Rightarrow\boxed{x\neq\kappa\pi-\frac{5\pi}{3}}$

α) $\sigma\varphi\left(\frac{\pi}{2}-x+\frac{\pi}{4}\right)=\sigma\varphi\left(x+\frac{5\pi}{3}\right)\Leftrightarrow\sigma\varphi\left(\frac{3\pi}{4}-x\right)=\sigma\varphi\left(x+\frac{5\pi}{3}\right)\Leftrightarrow$

$\Leftrightarrow\frac{3\pi}{4}-x=\kappa\pi+x+\frac{5\pi}{3}\Leftrightarrow-2x=\kappa\pi+\frac{5\pi}{3}-\frac{3\pi}{4}\Leftrightarrow-2x=\kappa\pi+\frac{20\pi-9\pi}{12}$

$\Leftrightarrow x=-\frac{\kappa\pi}{2}-\frac{11\pi}{24}\quad\kappa\in\mathbb{Z}$

β) $\sigma\upsilon\nu\left(2x+\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}\Leftrightarrow\sigma\upsilon\nu\left(2x+\frac{\pi}{6}\right)=\sigma\upsilon\nu\left(\frac{5\pi}{6}\right)\Leftrightarrow 2x=2\kappa\pi\pm\frac{5\pi}{6}-\frac{\pi}{6},\ \kappa\in\mathbb{Z}$

$\boxed{x=\kappa\pi+\frac{\pi}{3}}$

$-\pi<x<\pi$

$-4/3<\kappa<2/3$

$\boxed{x=\kappa\pi-\frac{\pi}{2}}$

$-\pi<x<\pi$

$-\frac{1}{2}<\kappa<\frac{3}{2}$

Δηλ. $x=-\frac{2\pi}{3}$ ή $x=\pi/3$

$x=-\frac{\pi}{2}$ ή $x=\frac{\pi}{2}$

Δ4

$$\sigma\varphi\frac{3\pi}{7}\cdot\sigma\varphi\frac{\pi}{14}+\eta\mu^2\frac{\pi}{7}+\eta\mu^2\frac{5\pi}{14}=$$

$$=\varepsilon\varphi\frac{\pi}{14}\cdot\sigma\varphi\frac{\pi}{14}+\eta\mu^2\frac{\pi}{7}+\sigma\upsilon\nu^2\frac{\pi}{7}=1+1=2$$