ΛΥΣΕΙΣ ΔΙΑΓ. Β ΛΥΚΕΙΟΥ ΑΛΓΕΒΡΑ

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ΘΕΜΑ Α

$\boxed{A3}$ 1. Λ 2. Λ 3. Λ 4. Σ 5. Σ 6 Σ

ΘΕΜΑ Β

$\boxed{B1}$ (1) $\Delta = 1+8 = 9$ άρα $\eta\mu x = \begin{cases} \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2} \\ \frac{-1-3}{4} = \frac{-4}{4} = -1 \end{cases}$

- $\eta\mu x = \frac{1}{2} \Leftrightarrow \eta\mu x = \eta\mu \frac{\pi}{6}$

  άρα $x = 2\kappa\pi + \frac{\pi}{6}$ ή $x = 2\kappa\pi + \frac{5\pi}{6}$ $\kappa \in \mathbb{Z}$

- $\eta\mu x = -1 \Leftrightarrow \eta\mu x = \eta\mu\left(-\frac{\pi}{2}\right)$ άρα $x = 2\kappa\pi - \frac{\pi}{2}$ ή $x = 2\kappa\pi + \frac{3\pi}{2}$, $\kappa \in \mathbb{Z}$

(2) $\eta\mu x - \sigma\upsilon\nu x = 0 \Leftrightarrow \varepsilon\varphi x - 1 = 0 \Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{4} \Leftrightarrow x = \kappa\pi + \frac{\pi}{4}$, $\kappa \in \mathbb{Z}$

(3) Πρέπει: • $2x \neq \kappa\pi + \frac{\pi}{2} \Rightarrow x \neq \frac{\kappa\pi}{2} + \frac{\pi}{4}$

• $\frac{\pi}{3} + 3x \neq \kappa\pi \Rightarrow 3x \neq \kappa\pi - \frac{\pi}{3} \Rightarrow x \neq \frac{\kappa\pi}{3} - \frac{\pi}{9}$, $\kappa \in \mathbb{Z}$

- $\varepsilon\varphi(2x) = \sigma\varphi\left(\frac{\pi}{3} + 3x\right) \Leftrightarrow \sigma\varphi\left(\frac{\pi}{2} - 2x\right) = \sigma\varphi\left(\frac{\pi}{3} + 3x\right)$

$\Leftrightarrow \frac{\pi}{2} - 2x = \kappa\pi + \frac{\pi}{3} + 3x \Leftrightarrow -5x = \kappa\pi + \frac{\pi}{3} - \frac{\pi}{2}$

$\Leftrightarrow x = -\frac{\kappa\pi}{5} + \frac{\pi}{30}$ $\kappa \in \mathbb{Z}$

$\boxed{B2}$ • $\sigma\upsilon\nu\left(\frac{4\pi}{3}\right) = \sigma\upsilon\nu\left(\pi + \frac{\pi}{3}\right) = -\sigma\upsilon\nu\frac{\pi}{3} = -\frac{1}{2}$ άρα $\rho_1 = -1$

- $\varepsilon\varphi\left(\frac{5\pi}{4}\right) = \varepsilon\varphi\left(\pi + \frac{\pi}{4}\right) = \varepsilon\varphi\frac{\pi}{4} = 1$ άρα $\rho_2 = 3$

$\begin{cases} P(\rho_1) = 0 \\ P(\rho_2) = 0 \end{cases} \Leftrightarrow \begin{cases} -3 + \lambda - \mu - 6 = 0 \Leftrightarrow \lambda - \mu = 9 \\ 81 + 9\lambda + 3\mu - 6 = 0 \Leftrightarrow 9\lambda + 3\mu = -75 \end{cases} \Leftrightarrow \begin{cases} \lambda - \mu = 9 \\ 3\lambda + \mu = -25 \end{cases}$

$\Leftrightarrow 4\lambda = -16 \Leftrightarrow \boxed{\lambda = -4}$ κ' $\boxed{\mu = 5}$

ΘΕΜΑ Γ

Γ1

- $\lambda^3 - 16\lambda = 0 \Leftrightarrow \lambda(\lambda^2 - 16) = 0 \Leftrightarrow \lambda = 0$ ή $\lambda = \pm 4$
- Αν $\lambda \neq 0, \pm 4 \rightarrow 3^{ου}$ ΒΑΘΜΟΥ
- Αν $\lambda = 0$: $P(x) = -16x + 4 \rightarrow 1^{ου}$ ΒΑΘΜΟΥ
- Αν $\lambda = 4$: $P(x) = 32x^2 + 8 \rightarrow 2^{ου}$ ΒΑΘΜΟΥ
- Αν $\lambda = -4$: $P(x) = 0 \rightarrow$ ΔΕΝ ΕΧΕΙ ΒΑΘΜΟ

Γ2 ①

- $P(-1) = 0 \Leftrightarrow \alpha + \beta + (\beta - 1) + \beta = 0$

$\Leftrightarrow \alpha + 3\beta = 1$

- $Q(-1) = 0 \Leftrightarrow -\beta + 1 + \cancel{\beta} - 2\alpha + \alpha - \cancel{\beta} = 0 \Leftrightarrow -\alpha - \beta = -1$

$$\begin{cases} \alpha + 3\beta = 1 \\ -\alpha - \beta = -1 \end{cases} + \Leftrightarrow 2\beta = 0 \Leftrightarrow \boxed{\beta = 0} \text{ κ' } \boxed{\alpha = 1}$$

② $P(x) = x^2 + x$ ΚΑΙ $Q(x) = -x^3 - 2x^2 - x$

$A(x) = (2x)^2 + (2x) - (x-1)^2 - (x-1) - 3x^2 + 3$

$= 4x^2 + 2x - x^2 + 2x - 1 - x + 1 - 3x^2 + 3 = \boxed{3x + 3} \rightarrow 1^{ου}$ ΒΑΘΜΟΥ

③ $(x+3)B(x) = -x^3 - 2x^2 - x - 12 \Leftrightarrow (x+3)(\alpha x^2 + \beta x + \gamma) = -x^3 - 2x^2 - x - 12$

$\Leftrightarrow \alpha x^3 + \beta x^2 + \gamma x + 3\alpha x^2 + 3\beta x + 3\gamma = -x^3 - 2x^2 - x - 12$

$\Leftrightarrow \alpha x^3 + (\beta + 3\alpha)x^2 + (\gamma + 3\beta)x + 3\gamma = -x^3 - 2x^2 - x - 12$

$\alpha = -1$

$\beta + 3\alpha = -2 \Leftrightarrow \beta = 1$

$\gamma + 3\beta = -1$

$\gamma = -4$

άρα $\boxed{B(x) = -x^2 + x - 4}$

Θέμα Δ

Δ1 $f(x) = \alpha\,συν2x - \sqrt{3}$

① $f\left(\frac{5\pi}{12}\right) = -2\sqrt{3} \Leftrightarrow \alpha\,συν2\frac{5\pi}{12} - \sqrt{3} = -2\sqrt{3}$

$\Leftrightarrow \alpha\,συν\frac{5\pi}{6} = -\sqrt{3} \Leftrightarrow \alpha\,συν\left(\pi - \frac{\pi}{6}\right) = -\sqrt{3} \Leftrightarrow \alpha\left(-\frac{\cancel{\sqrt{3}}}{2}\right) = -\cancel{\sqrt{3}}$

άρα $\boxed{\alpha = 2}$

② (α) $f(x) = 0 \Leftrightarrow 2συν2x - \sqrt{3} = 0 \Leftrightarrow συν2x = \frac{\sqrt{3}}{2} \Leftrightarrow συν2x = συν\frac{\pi}{6}$

$2x = 2κ\pi \pm \frac{\pi}{6} \Leftrightarrow \boxed{x = κ\pi \pm \frac{\pi}{12}}$ $κ \in \mathbb{Z}$

(β) $f\left(\frac{\pi}{2} - x\right) - f(x) = 2 \Leftrightarrow 2συν2\left(\frac{\pi}{2} - x\right) - 2συν2x = 2$

$\Leftrightarrow 2συν(\pi - 2x) - 2συν2x = 2 \Leftrightarrow -2συν2x - 2συν2x = 2$

$\Leftrightarrow -4συν2x = 2 \Leftrightarrow συν2x = -\frac{1}{2} \Leftrightarrow συν2x = συν\left(\pi - \frac{\pi}{3}\right)$

$\Leftrightarrow 2x = 2κ\pi \pm \frac{2\pi}{3} \Leftrightarrow \boxed{x = κ\pi \pm \frac{\pi}{3}}$, $κ \in \mathbb{Z}$

③ $-1 \le συν2x \le 1 \Leftrightarrow -2 \le 2συν2x \le 2 \Leftrightarrow -2-\sqrt{3} \le f(x) \le 2-\sqrt{3}$

• $f(x) = 2-\sqrt{3} \Leftrightarrow 2συν2x - \sqrt{3} = 2 - \sqrt{3} \Leftrightarrow συν2x = 1 \Leftrightarrow 2x = 2κ\pi \pm 0$

άρα $\boxed{x = κ\pi}$ $κ \in \mathbb{Z}$

Δ2 Έχουμε: $συν\left(\frac{\pi x - \pi}{4}\right) - x^4 + 2x^2 - 2 = 0 \Leftrightarrow συν\left(\frac{\pi x - \pi}{4}\right) = (x^2-1)^2 + 1$ (1)

Επειδή $συν\left(\frac{\pi x - \pi}{4}\right) \le 1$ τότε $(x^2-1)^2 + 1 \le 1 \Leftrightarrow (x^2-1)^2 \le 0$

$\Leftrightarrow x^2 - 1 = 0 \Leftrightarrow x = \pm 1$

• Για $x = 1$: $συν\left(\frac{\pi - \pi}{4}\right) = (1-1)^2 + 1 \Leftrightarrow συν0 = 1$, ισχύει

• Για $x = -1$: $συν\left(\frac{-\pi - \pi}{4}\right) = 0 + 1 \Leftrightarrow συν\left(-\frac{\pi}{2}\right) = 1 \Leftrightarrow 0 = 1 \to$ Άτοπο

Άρα $\boxed{x = 1}$