Λύσεις διαγωνίσματος 21/11/2020

ΘΕΜΑ Α A_1-γ A_2-γ A_3-δ A_4-α A_5 ΛΛΣΛΣ

ΘΕΜΑ Β **Β1-α** $I_{ολ\Delta} = I_{AB} + I_{\Gamma\Delta} + I_{\delta\alpha}$

$$\Rightarrow I_{ολ\Delta} = \frac{1}{12}m\ell^2 + \frac{1}{12}m\ell^2 + m\frac{\ell^2}{4} + \left(mR^2 + m(\ell+R)^2\right)$$

$$\Rightarrow I_{ολ\Delta} = \frac{1}{6}m\ell^2 + \frac{1}{4}m\ell^2 + m\frac{\ell^2}{4} + \frac{9}{4}m\ell^2$$

$$\Rightarrow I_{ολ\Delta} = \left(\frac{1}{6} + \frac{1}{4} + \frac{10}{4}\right)m\ell^2 = \frac{2+33}{12}\ell^2 = \frac{35}{12}m\ell^2.$$

$$\Rightarrow \boxed{I_{ολ\Delta} = \frac{35}{12}m\ell^2}$$

Β2 I-β, II-β I) Χάνει επαφή στη ΘΦΜ $A = 2\Delta\ell$, $y = \Delta\ell = A/2$

ΘΙ $K\Delta\ell = 2mg$

II) ΑΔΕΤ στη ΘΦΜ: $\upsilon = \omega\sqrt{A^2 - y^2} \Rightarrow \upsilon^2 = \frac{K}{2m}\left(A^2 - \frac{A^2}{4}\right) = \frac{K}{2m}\frac{3}{4}A^2$

από ΘΙ $\frac{K}{2m} = \frac{g}{\Delta\ell} \rightarrow \upsilon^2 = \frac{g}{\Delta\ell}\frac{3}{4}A^2 \Rightarrow \upsilon^2 = \frac{3}{4}\frac{g}{A/2}A^2 \Rightarrow \upsilon^2 = \frac{3}{2}gA$

ΘΜΚΕ $0 - \frac{1}{2}m\upsilon^2 = -mg\cdot h \Rightarrow h = \frac{\upsilon^2}{2g} = \frac{\frac{3}{2}gA}{2g} \Rightarrow h = \frac{3}{4}A$

Άρα διανύει $d = A + \Delta\ell + h = A + \frac{A}{2} + \frac{3}{4}A \Rightarrow d = \frac{9}{4}A \Rightarrow \boxed{d = 2{,}25A}$

Β3 I-α II-γ $P_2 = P_{atm} + \rho_1 g H = P_{atm} + \rho_1 g\frac{P_{atm}}{10\rho_1 g} = P_{atm} + \frac{1}{10}P_{atm}$

$\Rightarrow P_2 = 1{,}1\,P_{atm}$ και $A_1\upsilon_1 = A_2\upsilon_2 \Rightarrow \upsilon_1 = \frac{\upsilon_2}{2} \Rightarrow \upsilon_2 = 2\upsilon_1$

Bernoulli $P_1 + \frac{1}{2}\rho_1\upsilon_1^2 = P_2 + \frac{1}{2}\rho_2\upsilon_2^2 \Rightarrow 1{,}6\,P_{atm} + \frac{1}{2}\rho_1\upsilon_1^2 = 1{,}1\,P_{atm} + \frac{1}{2}\rho_1 4\upsilon_1^2$

$1{,}6\,P_{atm} - 1{,}1\,P_{atm} = \frac{1}{2}\rho_1\upsilon_1^2(4-1) \Rightarrow 0{,}5\,P_{atm} = \frac{1}{2}\rho_1\upsilon_1^2\cdot 3 \Rightarrow \upsilon_1^2 = \frac{P_{atm}}{3\rho_1}$

$\Rightarrow \upsilon_1 = \sqrt{\frac{P_{atm}}{3\rho_1}}$

Αριστερά $P_A = P_1 + \rho_1 g h_1$ Δεξιά $\begin{cases} P_B = P_2 + \rho_1 g h_2 \\ P_\Gamma = P_B + \rho_2 g\Delta h \end{cases} \Longrightarrow P_A = P_\Gamma$

$\Rightarrow P_1 + \rho_1 g h_1 = P_2 + \rho_1 g h_2 + \rho_2 g\Delta h \Rightarrow P_1 - P_2 = \rho_2 g\Delta h - \rho_1 g(h_1 - h_2)$

$\Rightarrow 1{,}6\,P_{atm} - 1{,}1\,P_{atm} = \rho_2 g\Delta h - \rho_1 g\Delta h$

$\Rightarrow 0{,}5\,P_{atm} = 5\rho_1 g\Delta h - \rho_1 g\Delta h \Rightarrow \frac{1}{2}P_{atm} = 4\rho_1 g\cdot\Delta h \quad \left(H = \frac{P_{atm}}{10\rho_1 g}\right)$

$\Rightarrow \frac{1}{2}10\rho_1 g H = 4\rho_1 g\Delta h \Rightarrow \boxed{\Delta h = 1{,}25H}$

ΘΕΜΑ Γ

$H = 5m \quad u = 5m/s \quad A = 2\cdot 10^{-3} m^3 \quad h = 4m \quad V = 20 m^3 \quad d = 3{,}2m \quad A_1 = 4\cdot 10^{-3} m^2 \quad A_2 = 2\cdot 10^{-3} m^2$

Γ1 α) $\Pi = \frac{V}{t} \Rightarrow A\cdot u = \frac{V}{t} \Rightarrow t = \frac{V}{Au} = \frac{36}{10^{-2}} = 3600\ sec \Rightarrow \boxed{t = 3600\ sec = 1h}$

β) ΘΜΚΕ $\frac{1}{2}\Delta m u^2 - 0 = -\Delta m g H + W_{αντ} \Rightarrow W_{αντ} = \frac{1}{2}\Delta m u^2 + \Delta m g H$

$W_{αντ} = \frac{1}{2}\rho\cdot\Delta V\cdot u^2 + \rho\cdot\Delta V g H \qquad \Delta V = V = 36 m^3.$

$W_{αντ} = \frac{1}{2}10^3\cdot 36\cdot 25 + 10^3\cdot 36\cdot 10\cdot 5 = 450\cdot 10^3 + 1800\cdot 10^3 = 2200\cdot 10^3 J$

$\boxed{W_{αντ} = 22{,}5\cdot 10^5 J}$

γ) $P_{αντ} = \frac{W_{αντ}}{\Delta t} = \frac{1}{2}\rho\Pi u^2 + \rho\Pi g H = \rho\Pi\left(\frac{1}{2}u^2 + gH\right)$, $\Pi = Au = 10^{-2} m^3/s.$

$P_{αντ} = 10^3\cdot 10^{-2}(12{,}5 + 50) \Rightarrow P_{αντ} = 62{,}5\cdot 10 \Rightarrow \boxed{P_{αντ} = 625 W}$

Γ2 $\Pi = Au = 10^{-2} m^3/s = \Pi_{αντλίας}$

$u_2 = \sqrt{2g(h-d)} = \sqrt{2\cdot 10\cdot 0{,}8} \Rightarrow u_2 = 4 m/s \qquad \Pi_2 = \Pi_{οπής} = A_2 u_2 = 8\cdot 10^{-3} m^3/s.$

$\Pi_{αντλίας} > \Pi_{οπής} \rightarrow$ η στάθμη ανεβαίνει

Όταν σταθεροποιείται $\Pi_{αντλίας} = \Pi'_{οπής} \Rightarrow 10^{-2} = A_2 u_2' \Rightarrow 10^{-2} = 2\cdot 10^{-3} u_2'$

$\Rightarrow u_2' = 5 m/s. \rightarrow u_2' = \sqrt{2g(h'-d)} \Rightarrow u_2'^2 = 2g(h'-d) \Rightarrow 25 = 20(h'-d)$

$1{,}25 = h' - 3{,}2 \Rightarrow \boxed{h' = 4{,}45m}$

Γ3 $u_2' = 5m/s \qquad x_{max} = u_2'\cdot t_{εδ} = 5\cdot 0{,}8 \Rightarrow \boxed{x_{max} = 4m}$

$t_{εδ} = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2\cdot 3{,}2}{10}} = \sqrt{0{,}64} = 0{,}8\ sec$

Γ4 $P_1 + \frac{1}{2}\rho u_1'^2 = P_{atm} + \frac{1}{2}\rho u_2'^2 \qquad A_1 u_1' = A_2 u_2' \Rightarrow 25 u_1' = 20 u_2'$

$u_1' = \frac{4}{5}u_2' = 4 m/s.$

$P_1 = P_{atm} + \frac{1}{2}\rho(u_2'^2 - u_1'^2)$

$P_1 = P_{atm} + \frac{1}{2}\rho\left(u_2'^2 - \frac{16}{25}u_2'^2\right) = P_{atm} + \frac{1}{2}\rho\frac{9}{25}u_2'^2$

$P_1 = 100\cdot 10^3 + \frac{9}{50}\cdot 25\cdot 10^3 = 100\cdot 10^3 + 4{,}5\cdot 10^3 = 104{,}5\cdot 10^3 N/m^2$

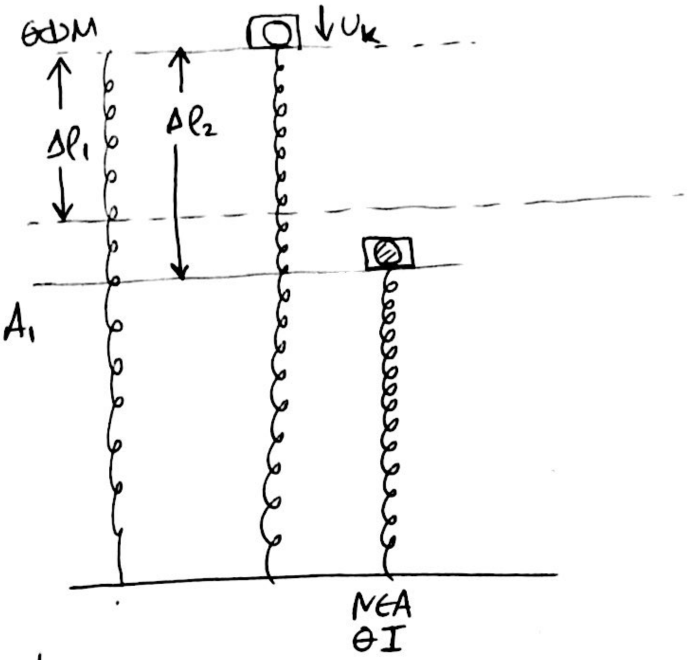
$\boxed{P_1 = 1{,}045\cdot 10^5 N/m^2}$

$P = \frac{W}{\Delta t} = \frac{(P_1 - P_2)\Delta V}{\Delta t} = (P_1 - \underset{P_{atm}}{P_2})\underset{A_2 u_2' = 10^{-2} m^3/s}{\Pi} = (104{,}5\cdot 10^3 - 100\cdot 10^3)10^{-2} \Rightarrow \boxed{P = 45 W}$

ΘΕΜΑ Δ

$m_1 = 3kg \quad k = 100 N/m \quad m_2 = 1kg$

$υ_0 = \sqrt{3}\, m/s \quad υ_K = 2 m/s$



Δ1 α) $ω_1 = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}} = \frac{\sqrt{3}}{3}10$

$υ_{1max} = υ_0 = ω_1 A_1 \Rightarrow \sqrt{3} = \frac{10}{\sqrt{3}} A_1 \Rightarrow \boxed{A_1 = 0,3m}$

β) ΘΙ m_1 $\quad kΔl_1 = m_1 g \Rightarrow Δl_1 = \frac{m_1 g}{k} = 0,3m = A_1$

Άρα η ΘΦΜ = άνω ακραία $υ_1 = 0$.

ΑΔΟ $\quad m_2 υ_2 + \cancel{m_1 υ_1} = m_{ολ} υ_K$

$1 \cdot υ_2 = 4 \cdot 2\sqrt{3} \quad \boxed{υ_2 = 8\sqrt{3}\, m/s}$

γ) $h_2 = \frac{υ_2^2 - υ^2}{2g} = \frac{64\cdot 3 - 36\cdot 3}{2\cdot 10} = \frac{3\cdot 28}{2\cdot 10} \Rightarrow h_2 = 4,2m$ ------> $h = h_2 + Δl_1 = 4,5m$

Δ2 ΝΕΑ ΘΙ $\quad m_{ολ} g = kΔl_2 \Rightarrow Δl_2 = \frac{m_{ολ} g}{k} = 0,4m$

ΑΔΕΤ $\quad \frac{1}{2}kA^2 = \frac{1}{2}m_{ολ}υ_K^2 + \frac{1}{2}kΔl_2^2 \Rightarrow A = \sqrt{\frac{m_{ολ}}{k}υ_K^2 + Δl_2^2} = \sqrt{\frac{4}{100}\cdot 12 + \frac{16}{100}}$

$\Rightarrow A = \sqrt{\frac{64}{100}} \Rightarrow \underline{A = 0,8m}$ $\quad t=0 \quad y = +0,4m \quad υ<0 \quad y = +A/2$

$φ_0 = 5π/6 \quad ω = \sqrt{\frac{k}{m_{ολ}}} = 5 rad/s$

$y = Aημ(ωt + φ_0)$

$\boxed{y = 0,8 \cdot ημ(5t + 5π/6)\ SI}$

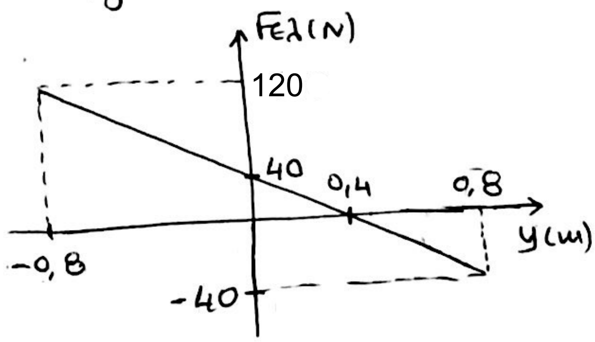
Δ3 $ΣF = m_{ολ}α \Rightarrow F_{ελ} - m_{ολ}g = -m_{ολ}ω^2 y \Rightarrow F_{ελ} = m_{ολ}g - k\cdot y$

$\Rightarrow \boxed{F_{ελ} = 40 - 100\cdot y} \quad -0,8m \le y \le +0,8m$

$y=0 \quad F_{ελ} = 40N, \quad y = +0,8m \rightarrow F_{ελ} = -40N$

$y = -0,8m \rightarrow F_{ελ} = 120N$

$y = +0,4m \rightarrow F_{ελ} = 0$



Δ4 $\left|\frac{dP_1}{dt}\right| = ΣF_1 = m_1|α| = m_1 ω^2 |y| \Rightarrow 30 = 3\cdot 25\cdot |y|$

$\Rightarrow |y| = 0,4m \rightarrow |υ| = ω\sqrt{A^2 - y^2} = 5\sqrt{\frac{64}{100} - \frac{16}{100}} = \frac{5\cdot\sqrt{48}}{10} = \frac{4\sqrt{3}}{2} \Rightarrow |υ| = 2\sqrt{3}\, m/s.$

προς τα κάτω = προς τα αρνητικά $\quad υ = -2 m/s$ $\quad \Big\} \quad \frac{dK}{dt} = ΣFυ = -k\cdot y\cdot υ.$

↳ δύο θέσεις $y = \pm 0,4m$.

πάνω από ΘΙ $y = +0,4m, υ = -2\sqrt{3}\, m/s \rightarrow \frac{dK}{dt} = -100\cdot(+0,4)(-2\sqrt{3}) \Rightarrow \boxed{\frac{dK}{dt} = +80\sqrt{3}\,\frac{J}{s}}$

κάτω από ΘΙ $y = -0,4m, υ = -2\sqrt{3}\, m/s \rightarrow \frac{dK}{dt} = -100(-0,4)(-2\sqrt{3}) \Rightarrow \boxed{\frac{dK}{dt} = -80\sqrt{3}\,\frac{J}{s}}$