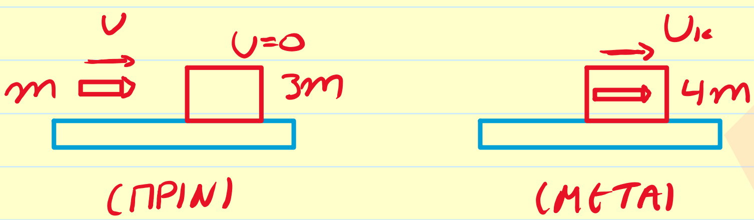
ΘΕΜΑ Α

Α1 β **Α2** α **Α3** δ **Α4** δ

Α5 α) Σ β) Σ γ) Σ δ) Λ ε) Σ

ΘΕΜΑ Β

Β1



Εφαρμόζουμε την Α.Δ.Ο.:

$\vec{p}_1 + \vec{p}_2 = \vec{p}_{κοινη} \overset{(+)}{\Rightarrow} m \cdot υ + 0 = (m+3m) \cdot υ_κ$

$\Rightarrow m \cdot υ = 4m \cdot υ_κ \Rightarrow υ_κ = \frac{υ}{4}$

$p_1 = m \cdot υ$

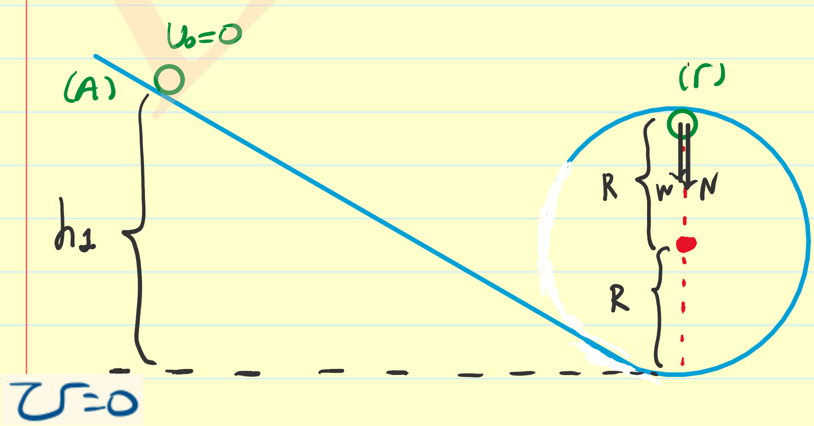
$p_1' = m \cdot υ_κ = m \cdot \frac{υ}{4}$

$\Rightarrow \Delta\vec{p}_1 = \vec{p}_1' - \vec{p}_1 \overset{(+)}{\Rightarrow} \Delta p_1 = m \frac{υ}{4} - mυ$

$\Rightarrow \Delta p_1 = -\frac{3}{4} m \cdot υ$

$\rightsquigarrow |\Delta p_1| = \frac{3}{4} m υ \rightsquigarrow$ (α)

Β2



Στη θέση (Γ):

$\Sigma F_R = N + mg$

$\Rightarrow m \cdot \frac{υ_Γ^2}{R} = N + mg$

$\Rightarrow N = m \cdot \frac{υ_Γ^2}{R} - mg$

~> Για να εκτελέσει με ασφάλεια ανακύκλωση:

$$N \geq 0 \Rightarrow m \cdot \frac{U_\Gamma^2}{R} - mg \geq 0 \Rightarrow U_\Gamma^2 \geq g \cdot R \Rightarrow U_\Gamma \geq \sqrt{gR}$$

Στην οριακή περίπτωση: $U_\Gamma = \sqrt{gR}$

~> <u>Α.Δ.Μ.Ε. (Α→Γ)</u>

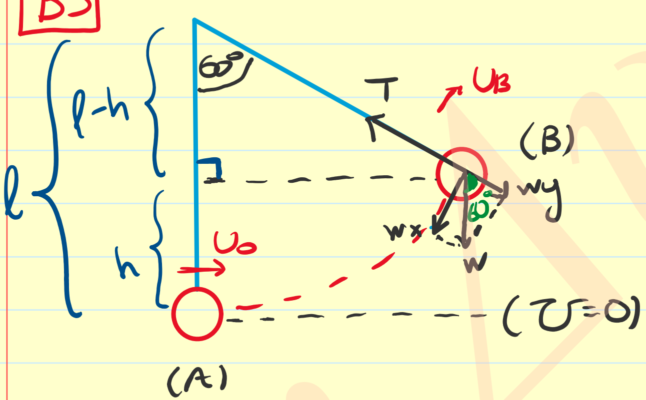
$$K_A + U_A = K_\Gamma + U_\Gamma \Rightarrow m g \cdot h_1 = \frac{1}{2} m \cdot U_\Gamma^2 + mg\, 2R$$

$$\Rightarrow g \cdot h_1 = \frac{1}{2} \cdot (\sqrt{gR})^2 + g \cdot 2R$$

$$\Rightarrow g \cdot h_1 = \frac{1}{2} \cdot g R + 2g \cdot R$$

$$\Rightarrow h_1 = \frac{R}{2} + 2R \Rightarrow \boxed{h_1 = \frac{5R}{2}}$$ ~> (γ)

B3



$$\eta\mu 60° = \frac{w_x}{w} \Rightarrow w_x = w \cdot \eta\mu 60°$$

$$\sigma\upsilon\nu 60° = \frac{w_y}{w} \Rightarrow w_y = w \cdot \sigma\upsilon\nu 60°$$

$$\cdot\ \sigma\upsilon\nu 60° = \frac{\ell - h}{\ell} \Rightarrow \frac{1}{2} = \frac{\ell - h}{\ell}$$

$$\Rightarrow \ell = 2\ell - 2h \Rightarrow 2h = \ell$$

$$\Rightarrow h = \frac{\ell}{2}$$

~> <u>Α.Δ.Μ.Ε. (Α→Β)</u>

$$K_A + U_A = K_B + U_B \Rightarrow \frac{1}{2} m \cdot U_0^2 = \frac{1}{2} m\, U_B^2 + m \cdot g \cdot h$$

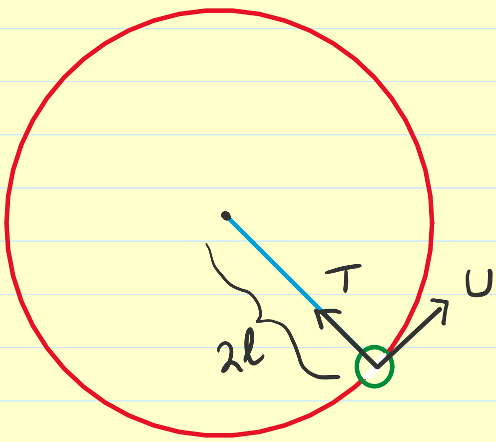
$$\Rightarrow \frac{(\sqrt{8g\ell})^2}{2} = \frac{U_B^2}{2} + g \cdot \frac{\ell}{2} \Rightarrow 8g\ell - g \cdot \ell = U_B^2$$

$$\Rightarrow U_B = \sqrt{7g\ell}$$

$\rightsquigarrow \Sigma F_R = T - w_y \Rightarrow m\cdot\frac{u_B^2}{\ell} = T - mg\cdot\sigma\upsilon\nu 60^\circ$

$\Rightarrow m\cdot\frac{(\sqrt{7g\ell})^2}{\ell} = T - mg\cdot\frac{1}{2} \Rightarrow 7mg + \frac{mg}{2} = T$

$\Rightarrow T = \frac{15}{2}mg \quad (1)$



Στο οριζόντιο δάπεδο:

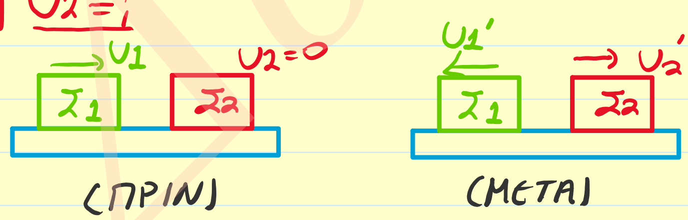
$\Sigma F_R = T = m\cdot\frac{u^2}{2\ell} \overset{(1)}{\Rightarrow} \frac{15}{2}mg = \frac{m\cdot u^2}{2\ell}$

$\Rightarrow \boxed{u = \sqrt{15 g\ell}}$ $\rightsquigarrow$ (β)

ΘΕΜΑ Γ

$m_1 = 2\,kg$
$u_1 = 5\,m/s$
$m_2 = 4\,kg$
$u_1' = 1\,m/s$
$\mu = 0{,}3$

Γ1 $u_2' = ;$



Α.Δ.Ο.

$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' \overset{(+)}{\Rightarrow} m_1 u_1 + 0 = -m_1 u_1' + m_2 u_2'$

$\Rightarrow 10 = -2 + 4\cdot u_2' \Rightarrow 4\cdot u_2' = 12 \Rightarrow \boxed{u_2' = 3\,m/s}$

Γ2 $\Sigma F_1 = ;$, αν $\Delta t = 0{,}01\,s$

$\Sigma \vec{F}_1 = \frac{\Delta \vec{p}_1}{\Delta t} \overset{(+)}{\Rightarrow} \Sigma F_1 = \frac{-m_1 u_1' - m_1 u_1}{\Delta t} = \frac{-2-10}{0{,}01} = \frac{-12}{0{,}01}$

$\Rightarrow \boxed{\Sigma F_1 = -1200\,N}$

Γ3 π=;

$K_{ολ,πριν} = K_1 = \frac{1}{2} m_1 \cdot υ_1^2 = \frac{1}{2} \cdot 2 \cdot 5^2 = 25J$

$K_{ολ,μετά} = K_1' + K_2' = \frac{1}{2} m_1 \cdot υ_1'^2 + \frac{1}{2} m_2 \cdot υ_2'^2$

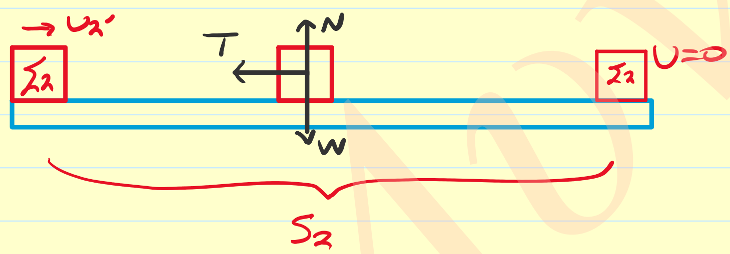
$= \frac{1}{2} \cdot 2 \cdot 1^2 + \frac{1}{2} 4 \cdot 3^2 = 1 + 18 = 19 J$

$E_{απωλειών} = K_{ολ,πριν} - K_{ολ,μετά} = 25 - 19 = 6J$

$\left.\begin{array}{l} K_{ολ,πριν} \longrightarrow E_{απωλειών} \\ 100\% \longrightarrow π; \end{array}\right\} \Rightarrow π = \frac{E_{απωλειών}}{K_{ολ,πριν}} \cdot 100\%$

$\Rightarrow π = \frac{6}{25} \cdot 100\% \Rightarrow \boxed{π = 24\%}$

Γ4 $Q_{τριβής}$ =;



$ΣF_y = 0 \Rightarrow N = m_2 g = 40N$

$T = μ \cdot N = 0,3 \cdot 40 = 12N$

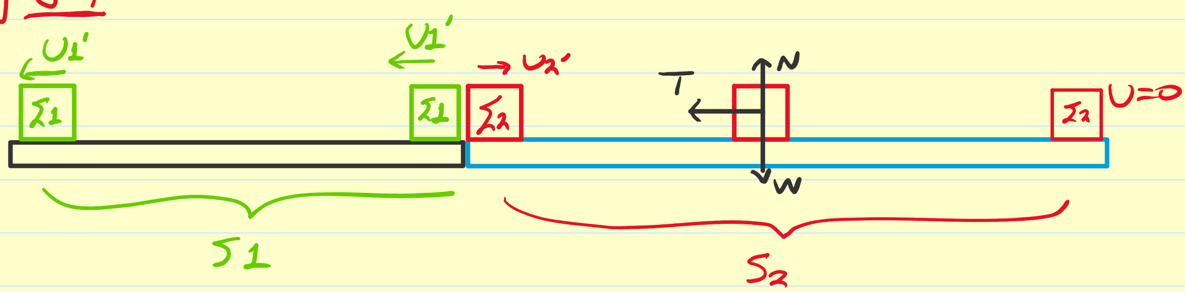
Θ.Μ.Κ.Ε.

$K_{τελ}^{\,0} - K_{αρχ} = W_T + W_w^{\,0} + W_N^{\,0} \Rightarrow -\frac{1}{2} m_2 \cdot υ_2'^2 = -T \cdot S_2$

$\Rightarrow -\frac{1}{2} \cdot 4 \cdot 3^2 = -12 \cdot S_2 \Rightarrow 18 = 12 \cdot S_2 \Rightarrow S_2 = 1,5m$

Άρα: $\boxed{Q_{τριβής} = |W_T| = |-T \cdot S_2| = 18 J}$

Γ5 d=;



- Υπολογίζουμε το χρόνο μέχρι να σταματήσει το Σ2:

$\Sigma \vec{F}_x = m_2 \cdot \vec{a} \Rightarrow T = m_2 \cdot a \Rightarrow 12 = 4 \cdot a \Rightarrow a = 3 m/s^2$

$υ = υ_2' - |a| \cdot \Delta t \Rightarrow 0 = 3 - 3 \cdot \Delta t \Rightarrow \Delta t = 1s$

- Στον ίδιο χρόνο, το Σ1 έχει διανύσει:

$S_1 = υ_1' \cdot \Delta t = 1m$

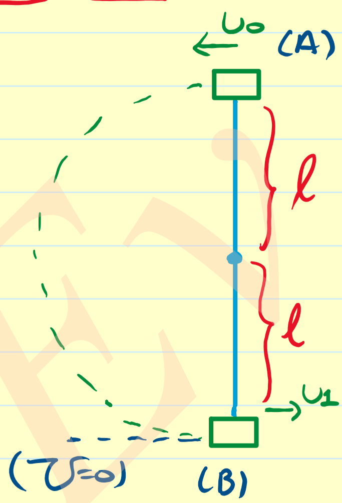
Άρα η μεταξύ τους απόσταση, είναι:

$d = S_1 + S_2 = 2,5m$

ΘΕΜΑ Δ

$m_1 = 2kg$, $\ell = 0,4m$, $υ_0 = 3m/s$, $H = 0,2m$, $m_2 = 8kg$, $υ_2' = 2m/s$

Δ1 $υ_1 = ;$



Α.Δ.Μ.Ε. (Α→Β)

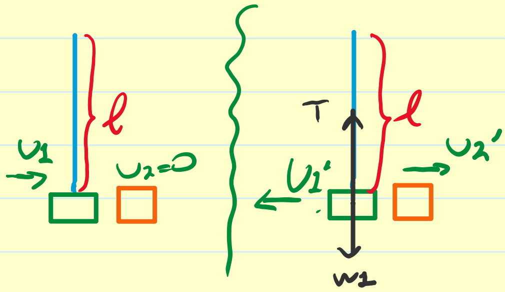
$K_A + U_A = K_B + U_B^{\nearrow 0}$

$\Rightarrow \frac{1}{2} m_1 \cdot υ_0^2 + m_1 g \cdot 2\ell = \frac{1}{2} m_1 \cdot υ_1^2$

$\Rightarrow 3^2 + 4 \cdot 10 \cdot 0,4 = υ_1^2$

$\Rightarrow υ_1^2 = 9 + 16 = 25 \Rightarrow υ_1 = 5m/s$

Δ2 $T = ;$



Α.Δ.Ο.

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1\,' + \vec{p}_2\,'$$

$$\overset{(+)}{\Rightarrow} m_1 \cdot υ_1 + 0 = m_1 \cdot υ_1' + m_2 \cdot υ_2'$$

$$\Rightarrow 2 \cdot 5 = 2 \cdot υ_1' + 8 \cdot 2$$

$$\Rightarrow 10 - 16 = 2 \cdot υ_1' \Rightarrow 2υ_1' = -6 \Rightarrow υ_1' = -3\,m/s$$

$$\sim \Sigma F_R = m_1 \frac{υ_1'^2}{\ell} = 2 \cdot \frac{3^2}{0{,}4} = \frac{18}{0{,}4} = 45N$$

$$\sim \Sigma F_R = T - m_1 g \Rightarrow T = \Sigma F_R + mg \Rightarrow \boxed{T = 65N}$$

Δ3 $π = ;$

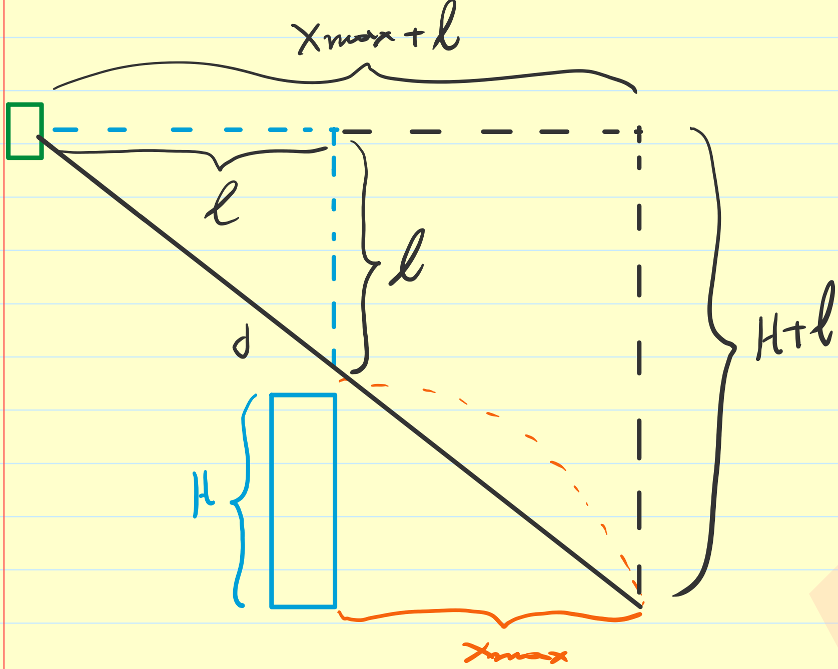
$$K_1 = \frac{1}{2} m_1 \cdot υ_1^2 = \frac{1}{2} \cdot 2 \cdot 5^2 = 25J$$

$$K_2' = \frac{1}{2} m_2 \cdot υ_2'^2 = \frac{1}{2} \cdot 8 \cdot 2^2 = 16J$$

$$\left.\begin{array}{l} K_1 \rightarrow K_2' \\ 100\% \rightarrow π; \end{array}\right\} \Rightarrow π = \frac{K_2'}{K_1} \cdot 100\% \Rightarrow π = \frac{16}{25} \cdot 100\%$$

$$\Rightarrow \boxed{π = 64\%}$$

Δ4 $d = ;$



$$\sim t_{εσ} = \sqrt{\frac{2H}{g}} = \sqrt{\frac{0{,}4}{10}} = 0{,}2\,s$$

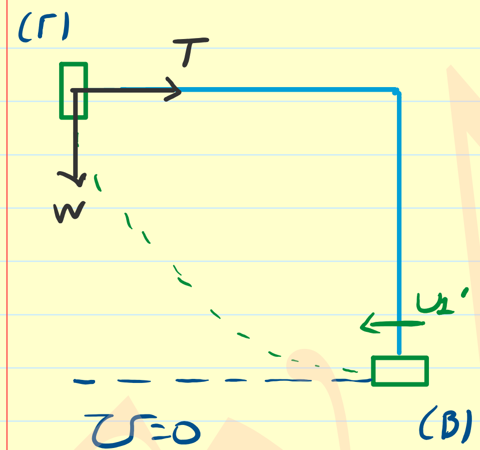
$$X_{max} = υ_1' \cdot t_{εσ} = 0{,}4\,m$$

$$\sim d = \sqrt{(X_{max}+\ell)^2 + (H+\ell)^2}$$

$$\Rightarrow d = \sqrt{0{,}8^2 + 0{,}6^2}$$

$$\Rightarrow \boxed{d = 1\,m}$$

Δ5 $\frac{Δp}{Δt} = ;$



Α.Δ.Μ.Ε.(Β→Γ)

$$K_B + \cancel{U_B}^{\,0} = K_Γ + U_Γ$$

$$\Rightarrow \frac{1}{2} m_1 υ_1'^2 = \frac{1}{2} m_1 υ_Γ^2 + m_1 g \ell$$

$$\Rightarrow 3^2 = υ_Γ^2 + 2 \cdot 10 \cdot 0{,}4$$

$$\Rightarrow υ_Γ = 1\,m/s$$

$$ΣF_R = m_1 \frac{υ_Γ^2}{\ell} = 2 \cdot \frac{1^2}{0{,}4} = 5N = T$$

$$w_1 = m_1 g = 20N$$

$$\Rightarrow \frac{Δp}{Δt} = ΣF = \sqrt{T^2 + w^2}$$

$$\Rightarrow \frac{Δp}{Δt} = \sqrt{5^2 + 20^2}$$

$$\Rightarrow \boxed{\frac{Δp}{Δt} = \sqrt{425}\,N}$$