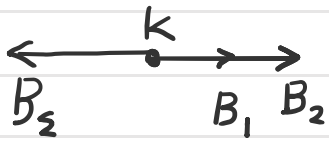
Λύσεις Διαγωνίσματος 28/2/2021

ΘΕΜΑ Α| $A_1-\alpha$, $A_2-\gamma$, $A_3-\gamma$, $A_4-\gamma$, A_5 ΛΑΘΟΣ

ΘΕΜΑ Β| **Β1-α**



$\vec{B}_{ολ_K} = \vec{0}$

$\vec{B}_\Sigma + \vec{B}_1 + \vec{B}_2 = \vec{0} \Rightarrow B_\Sigma = B_1 + B_2 \Rightarrow$

$\Rightarrow K_\mu 4\pi n I = \frac{2k_\mu I_1}{d} + \frac{2k_\mu I_2}{d} \Rightarrow \boxed{I = \frac{I_1 + I_2}{2\pi n d}}$

Β2-β Αρχικά $R_{ολ} = R + r = R + R/2 \Rightarrow R_{ολ} = 3R/2$

$I = \frac{\mathcal{E}}{R_{ολ}} = \frac{2\mathcal{E}}{3R}$ Ισορροπία $\Sigma F_y = 0 \Rightarrow F_L = mg \Rightarrow BI\ell = mg$ (1)

δ-κλείνω $R'_{ολ} = \frac{R \cdot R}{R+R} + r = \frac{R}{2} + \frac{R}{2} \Rightarrow R'_{ολ} = R$

$I' = \frac{\mathcal{E}}{R}$, $I' = I_{ΚΛ} + I_R$, $V_{ΚΛ} = V_R \Rightarrow I_{ΚΛ} R = I_R R \Rightarrow I_{ΚΛ} = I_R$

$I' = 2I_{ΚΛ}$

Άρα $I_{ΚΛ} = I'/2 = \frac{\mathcal{E}}{2R} \rightarrow F'_L = B I_{ΚΛ} \cdot \ell$

$\frac{I_{ΚΛ}}{I} = \frac{\mathcal{E}/2R}{2\mathcal{E}/3R} \Rightarrow \frac{I_{ΚΛ}}{I} = \frac{3}{4} \Rightarrow I_{ΚΛ} = \frac{3}{4} I$

$F'_L = B\frac{3}{4} I\ell = \frac{3}{4} BI\ell \xrightarrow{(1)} F'_L = \frac{3}{4} mg$

$\Sigma F_y = m\alpha \Rightarrow mg - F'_L = m \cdot \alpha \Rightarrow mg - \frac{3}{4} mg = m\alpha$

$\Rightarrow m\alpha = \frac{1}{4} mg \Rightarrow \boxed{\alpha = g/4 \downarrow}$

Β3-α ΑΔΟ: $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow \vec{P}_1 + \vec{P}_2 = \vec{P}_K$ $\downarrow +$

$P_1 - P_2 = 0 \Rightarrow m_1 v_1 = m_2 v_2 \Rightarrow 4m v_1 = m v_2 \Rightarrow v_2 = 4v_1$

$E_{απωλ}_{KP} = K_{πριν} - K_{μετα} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} 4m v_1^2 + \frac{1}{2} m 16 v_1^2$

$E_{απωλ}_{KP} = 2mv_1^2 + 8mv_1^2 \Rightarrow E_{απωλ}_{KP} = 10mv_1^2$

ΘΙ m_1 : $\Sigma F_1 = 0 \Rightarrow F_{ελ1} = m_1 g \Rightarrow k\Delta l_1 = 4mg \Rightarrow \Delta l_1 = \frac{4mg}{k}$

ΘΙ $m_{ολ}$: $\Sigma F = 0 \Rightarrow F_{ελ2} = m_{ολ} g \Rightarrow k\Delta l_2 = 5mg \Rightarrow \Delta l_2 = \frac{5mg}{k}$

$A_0 = \Delta l_2 - \Delta l_1 = \frac{5mg}{k} - \frac{4mg}{k} \Rightarrow A_0 = \frac{mg}{k}$

$A = A_0 e^{-\Lambda t}$ $\quad \Lambda t = \Lambda \frac{2\ln 2}{\Lambda} = 2\ln 2 = \ln 2^2 = \ln 4$

$A = A_0 e^{-\ln 4} = \frac{A_0}{e^{\ln 4}} \Rightarrow A = \frac{A_0}{4}$

$E_{απωλ}_{φθ} = E_0 - E$ όπου $E_0 = \frac{1}{2} D A_0^2$, $E = \frac{1}{2} D A^2 = \frac{1}{16}\frac{1}{2} D A_0^2$

$E_{απωλ}_{φθ} = \frac{1}{2} D A_0^2 - \frac{1}{16}\frac{1}{2} D A_0^2 = \frac{15}{32} D A_0^2$

$T_0 = T \rightarrow \omega_0 = \omega \rightarrow D = k$ $\quad E_{απωλ}_{φθ} = \frac{15}{32} k A_0^2$

Ισχύει $E_{απωλ}_{φθ} = \frac{1}{4} E_{απωλ}_{κρ} \Rightarrow \frac{15}{32} k A_0^2 = \frac{1}{4} 10 m v_1^2$

$\Rightarrow \frac{3}{16} k A_0^2 = m v_1^2 \Rightarrow v_1^2 = \frac{3}{16}\frac{k A_0^2}{m} \Rightarrow v_1^2 = \frac{3}{16}\frac{k}{m}\frac{m^2 g^2}{k^2}$

$\Rightarrow v_1^2 = \frac{3}{16}\frac{mg^2}{k} \Rightarrow \boxed{v_1 = \frac{g}{4}\sqrt{\frac{3m}{k}}}$

ΘΕΜΑ Γ

$A_1 = \sqrt{3}A$, $A_2 = A$ $\quad \varphi_{02} = \pi/2$ $\quad \Delta\varphi = \pi/2$

Γ1) Ισχύει: $x_{ολ} = x_1 + x_2 \Rightarrow x_{ολ} = \sqrt{3} A ημ(\omega t) + A ημ(\omega t + \frac{\pi}{2})$

$t = T/6$ $\quad x_{ολ} = \sqrt{3} A ημ\left(\frac{2\pi}{T}\frac{T}{6}\right) + A ημ\left(\frac{2\pi}{T}\frac{T}{6} + \frac{\pi}{2}\right)$

$\Rightarrow +0{,}4m = \sqrt{3} A ημ\frac{\pi}{3} + A ημ\frac{5\pi}{6}$

$\Rightarrow +0{,}4m = \sqrt{3} A \sqrt{3}/2 + A\, 1/2 \Rightarrow 2A = 0{,}4 \Rightarrow \boxed{A = 0{,}2m}$

$\Delta t = \frac{T}{2} \Rightarrow \frac{1}{100} = \frac{T}{2} \Rightarrow T = \frac{1}{50}\ sec$, $\boxed{\omega = \frac{2\pi}{T} = 100\pi \frac{rad}{sec}}$

Γ2 $A_{o\lambda} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \sigma\upsilon\nu\Delta\varphi^{\to 0}} = \sqrt{3A^2 + A^2} \Rightarrow A_{o\lambda} = 2A$

$\boxed{A_{o\lambda}\ \ 0,4m}$ $\quad E\varphi\varphi_0 = \frac{A_2 \eta\mu\Delta\varphi^{\to 1}}{A_1 + A_2 \sigma\upsilon\nu\Delta\varphi_{\to 0}} = \frac{A}{A\sqrt{3}} = \frac{\sqrt{3}}{3} \rightarrow \boxed{\varphi_0 = \frac{\pi}{6}}$

$x_{o\lambda} = A_{o\lambda}\, \eta\mu(\omega t + \varphi_0) \Rightarrow \boxed{x_{o\lambda} = 0,4 \cdot \eta\mu\left(100\pi t + \frac{\pi}{6}\right) SI}$

Γ3 $x_1 = -x_2$ $2^{η}$ φορά $\Rightarrow x_{o\lambda} = 0$ $\mu\epsilon$ $U > 0$

Αρα $U = U_{max} = \omega A_{o\lambda} = 100\pi \cdot 0,4 \Rightarrow \boxed{U = U_{max} = 40\pi \frac{m}{s}}$

Γ4 $\omega_1 = \omega = 100\pi \frac{rad}{s}$ $\omega_2 = \omega + \frac{4}{100}\omega = \frac{104}{100}\omega \Rightarrow \omega_2 = 104\pi \frac{rad}{s}$

$x'_{o\lambda} = 2A\, \sigma\upsilon\nu\left(\frac{\omega_1 - \omega_2}{2} t\right) \eta\mu\left(\frac{\omega_1 + \omega_2}{2} t\right) = 0,4 \cdot \sigma\upsilon\nu(-2t) \cdot \eta\mu(102t)$

$\boxed{x_{o\lambda} = 0,4 \cdot \sigma\upsilon\nu(2\pi t)\, \eta\mu(102\pi t)\ SI}$

Γ5 $U_1 = U_{1max}\, \sigma\upsilon\nu(\omega_1 t) = \omega_1 A\, \sigma\upsilon\nu(\omega_1 t) \Rightarrow U_1 = 20\pi\, \sigma\upsilon\nu(100\pi t)\ SI$

$U_2 = U_{2max}\, \sigma\upsilon\nu(\omega_2 t) = \omega_2 A\, \sigma\upsilon\nu(\omega_2 t) \Rightarrow U_2 = 20,8\pi\, \sigma\upsilon\nu(104\pi t)\ SI$

$T_S = \frac{2\pi}{|\omega_1 - \omega_2|} = \frac{2\pi}{4\pi} \Rightarrow T_S = \frac{1}{2}\ sec. \rightarrow t = T_S/4 = \frac{1}{8}\ sec$

$U = U_1 + U_2 = 20\, \sigma\upsilon\nu\left(100\frac{\pi}{8}\right) + 20,8\, \sigma\upsilon\nu\left(104\frac{\pi}{8}\right)$

$U = 20\pi\, \sigma\upsilon\nu(12,5\pi) + 20,8\pi\, \sigma\upsilon\nu(13\pi)$

$U = 20\pi\, \sigma\upsilon\nu(12\pi + \pi/2) + 20,8\pi\, \sigma\upsilon\nu(12\pi + \pi) = 20\pi\, \sigma\upsilon\nu\frac{\pi}{2}^{\to 0} + 20,8\pi\, \sigma\upsilon\nu\pi^{\to -1}$

$\boxed{U = -20,8\pi \frac{m}{s}}$

ΘΕΜΑ Δ

Δ1 ΘΜΚΕ για m_1: $K_{τελ} - K_{αρχ} = W_{w_1 g} \Rightarrow \frac{1}{2}m_1 v_1^2 - \frac{1}{2}m_1 v_0^2 = m_1 g \ell$

$\Rightarrow v_1^2 = v_0^2 + 2g\ell = 64 \Rightarrow v_1 = 8\,m/s.$

ΑΔΟ: $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow \vec{P}_1 = \vec{P}_1' + \vec{P}_2 \overset{+}{\Rightarrow} m_1 v_1 = 0 + m_2 v_2 \Rightarrow$

$\Rightarrow v_2 = \frac{m_1 v_1}{m_2} \Rightarrow v_2 = 16/3\ m/s.$

$K_{πριν} = \frac{1}{2}m_1 v_1^2 = 16\,J$

$K_{μετα} = \frac{1}{2}m_2 v_2^2 = \frac{32}{3}\,J$

$\Big\} \; E_{απωλ} = K_{πριν} - K_{μετα} = \frac{16}{3}\,J$

$\Pi_1 = \frac{E_{απωλ}}{K_{πριν}} 100\% \Rightarrow \boxed{\Pi_1 = 100/3\,\% = 33{,}33\,\%}$

Δ2 $v_2' = \frac{m_2 - m_3}{m_2 + m_3} v_2 \Rightarrow \boxed{v_2' = -\frac{4}{3}\,m/s}$, $v_3' = \frac{2m_2}{m_2 + m_3} v_2 \Rightarrow \boxed{v_3' = 4\,m/s}$

Δ3 $\Pi_2 = \frac{\Delta K_2}{K_2} 100\% = \frac{K_2' - K_2}{K_2} 100\% = \left(\frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_2 v_2^2} - 1\right) 100\%$

$\Pi_2 = \left[\left(\frac{v_2'}{v_2}\right)^2 - 1\right] 100\% = \left(\frac{1}{16} - 1\right) 100\% = -\frac{15}{16} 100\% \Rightarrow \boxed{\Pi = -93{,}75\%}$

Δ4 ΘΜΚΕ για $Σ_3$: $\cancel{K_{τελ}}^{0} - K_{3αρχ} = W_{T_3} + W_{F_{ελ}}$

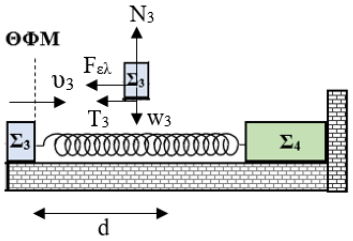
$0 - \frac{1}{2}m_3 v_3'^2 = -T_3 \cdot d - \frac{1}{2}k d^2$

όπου $T_3 = \mu N_3 = \mu m_3 g = 5\,N$

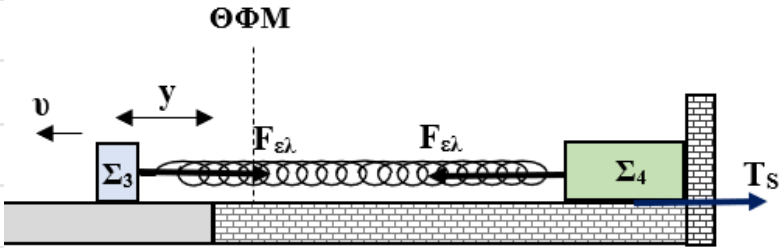
$-\frac{1}{2}\frac{5}{4} \cdot 16 = -5d - 50d^2 \Rightarrow$

$10d^2 + d - 2 = 0$

$\Delta = 81 \rightarrow \sqrt{\Delta} = 9 \quad d = \frac{-1 \pm 9}{20}$ $<$ $\boxed{d = 0{,}4\,m}$; $d = -0{,}5\,m$ απορ.



Δ5) Η ολίσθηση του Σ_4 αρχίζει όταν $F_{ελ} \geq T_{s\,max}$



$$F_{ελ} \geq T_{s\,max} \Rightarrow ky \geq \mu_s N_4 \Rightarrow ky \geq \mu_s m_4 g \Rightarrow y \geq \frac{\mu_s m_4 g}{k}$$

$$\Rightarrow y \geq \frac{0,4 \cdot 2,5\sqrt{2} \cdot 10}{100} \Rightarrow y \geq \frac{\sqrt{2}}{10}\,m \rightarrow y = 0,1\sqrt{2}\,m$$

Τότε $\frac{dP}{dt} = \Sigma F_x = F_{ελ} = ky \Rightarrow \boxed{\frac{dP}{dt} = 10\sqrt{2}\,N} \rightarrow$

ΘΜΚΕ για m_3: $K_{τελ}^{(y)} - K_{αρχ}^{ΘΦΜ} = W_{T_3}^{0\rightarrow 2d} + W_{Fελ}$

$$\frac{1}{2}m_3 v^2 - \frac{1}{2}m_3 v_3'^2 = -T_3 \cdot 2d + \cancel{U_{ελ\,αρχ}^{ΘΦΜ}}^{\,0} - U_{ελ\,τελ}^{(y)}$$

$$\frac{1}{2}m_3 v^2 - \frac{1}{2}m_3 v_3'^2 = -T_3 \cdot 2d - \frac{1}{2}ky^2$$

$$\frac{1}{2}\frac{5}{4}v^2 - \frac{1}{2}\frac{5}{4} \cdot 16 = -5 \cdot 0,8 - \frac{1}{2}100\frac{2}{100}$$

$$\frac{5}{8}v^2 = 10 - 4 - 1 \Rightarrow v^2 = 8 \Rightarrow \boxed{v = \sqrt{8}\,m/s = 2\sqrt{2}\,m/s}$$