Λύσεις διαγωνίσματος Φυσικής 16/4/2022

ΘΕΜΑ Α

Α1 - β Α2 - δ Α3 - α Α4 - α Α5 Λ Σ Λ Σ Λ

ΘΕΜΑ Β

Β1 | γ ΘΜΚΕ για m_1 στην κάθοδο: $\frac{1}{2}m_1 v_1^2 - 0 = m_1 g h \Rightarrow v_1^2 = 2gh$ (1)

Για κάθε σφαιρίδιο μετά την κρούση ΘΜΚΕ: $0 - \frac{1}{2}m v^2 = -mg\,\ell/6$

$\Rightarrow v^2 = \frac{1}{3}g\ell \Rightarrow v = \sqrt{\frac{g\ell}{3}} = v_1' = v_2'$ (2) καθώς μετά έχουν ίσες ταχύτητες

Άρα $|v_1'| = v_2' \Rightarrow \frac{|m_1 - m_2|}{m_1 + m_2} v_1 = \frac{2m_1}{m_1+m_2} v_1 \Rightarrow |m_1 - m_2| = 2m_1$

Επειδή $\vec{v}_1 \uparrow\downarrow \vec{v}_1'$ $m_1 < m_2$ άρα $-(m_1 - m_2) = 2m_1 \Rightarrow m_2 = 3m_1$

οπότε $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \Rightarrow v_1' = \frac{m_1 - 3m_1}{4m_1} v_1 \Rightarrow v_1' = -\frac{v_1}{2} \Rightarrow v_1'^2 = \frac{1}{4} v_1^2 \xRightarrow[(2)]{(1)}$

$\Rightarrow \frac{1}{3}g\ell = \frac{1}{4}2gh \Rightarrow \boxed{h = 2\ell/3}$ (γ)

Β2 | Ι - β | ΙΙ - α

Ι) ΑΔΣ $\underset{\text{πριν}}{\vec{L}_{(zz')}} = \underset{\text{μετά}}{\vec{L}_{(zz')}} \Rightarrow \vec{L}_{M(z)} + \cancel{\vec{L}_{m(z)}} = \vec{L}'_{M(z)} + \vec{L}'_{m(z)}$

$\Rightarrow L_{M(z)} = L'_{M(z)} + L'_{m(z)} \Rightarrow I_{cm} \cdot \omega = I_{cm}\omega_K + I_m \omega_K$

$\Rightarrow I_{cm}\omega = (I_{cm} + I_m)\omega_K \Rightarrow I_{cm}\omega = I_{ολ}\omega_K$

όπου $I_{ολ} = I_{cm} + I_m = \frac{1}{2}MR^2 + m r^2 = \frac{1}{2}MR^2 + \frac{M}{2}\frac{R^2}{4} = \frac{1}{2}MR^2 + \frac{1}{8}MR^2 = \frac{5}{8}MR^2$

$\Rightarrow \frac{1}{2}MR^2\omega = \frac{5}{8}MR^2\omega_K \Rightarrow \omega_K = \frac{4}{5}\omega$ $\Rightarrow \boxed{I_{ολ} = \frac{5}{4}I_{cm}}$

$\Pi = \frac{\Delta\omega}{\omega}100\% = \frac{\omega_K - \omega}{\omega}100\% = \left(\frac{\omega_K}{\omega} - 1\right)100\%$

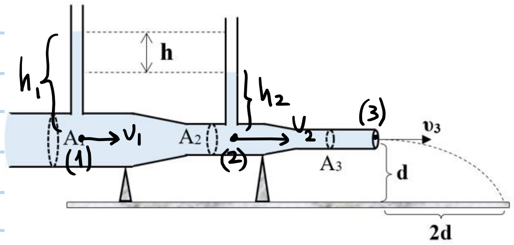
όμως $\omega_K = \frac{4}{5}\omega \Rightarrow \frac{\omega_K}{\omega} = \frac{4}{5}$

άρα $\Pi = \left(\frac{4}{5} - 1\right)100\% = -\frac{1}{5}100\% \Rightarrow \boxed{\Pi = -20\%}$ (β)

ΙΙ) $L'_{m(z)} = I_m\omega_K = m r^2\omega_K = \frac{M}{2}\frac{R^2}{4}\frac{4}{5}\omega = \frac{1}{5}\frac{1}{2}MR^2\omega = \frac{1}{5}I_{cm}\omega$

$\Rightarrow \boxed{L'_{m(z)} = \frac{1}{5}L_{M(z)}}$ (β)

Β3-α Ισχύει $A_1 v_1 = A_2 v_2 \Rightarrow 2A_2 v_1 = A_2 v_2 \Rightarrow v_2 = 2v_1$



Bernoulli $1 \to 2$: $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(4v_1^2 - v_1^2)$

$\Rightarrow P_1 - P_1 = \frac{3}{2}\rho v_1^2$ ① και $\left.\begin{aligned} P_1 &= P_{atm} + \rho g h_1 \\ P_2 &= P_{atm} + \rho g h_2 \end{aligned}\right\} \ominus \Rightarrow P_1 - P_2 = \rho g(h_1 - h_2) = \rho g h$ ②

$\overset{②}{\Rightarrow} \rho g h = \frac{3}{2}\rho v_1^2$

$\Rightarrow h = \frac{3}{2}\frac{v_1^2}{g}$ ③

Ισχύει: $x = v_3 t \Rightarrow 2d = v_3\sqrt{\frac{2d}{g}} \Rightarrow 4d^2 = v_3^2 \frac{2d}{g} \Rightarrow v_3^2 = 2gd$

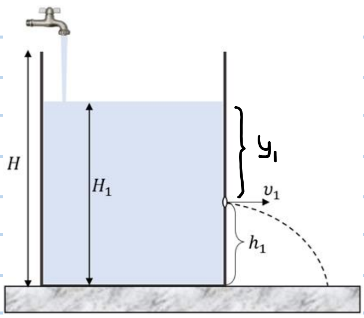
Επίσης $A_1 v_1 = A_3 v_3 \Rightarrow A_1 v_1 = \frac{A_1}{3} v_3 \Rightarrow v_1 = \frac{1}{3} v_3 \Rightarrow v_1^2 = \frac{1}{9} v_3^2 = \frac{2}{9} g d$ ④

③ $\overset{④}{\Rightarrow} h = \frac{3}{2}\frac{1}{g}\frac{2}{9} g d \Rightarrow \boxed{h = d/3}$ (α)

ΘΕΜΑ Γ

Γ1 Σταθερή στάθμη : $\Pi_{\varepsilon\rho} = \Pi_{o\pi\acute{\eta}\varsigma} = A \cdot v_1 \Rightarrow 2 \cdot 10^{-3} = 5 \cdot 10^{-4} v_1 \Rightarrow \boxed{v_1 = 4\,m/s}$

Γ2 Bernoulli από την επιφάνεια στην οπή:



$P_{atm\,\varepsilon\pi\iota\varphi} + 0 + \rho g y_1 = P_{atm\,o\pi\acute{\eta}\varsigma} + \frac{1}{2}\rho v_1^2 + 0 \Rightarrow v_1^2 = 2 g y_1$

$\Rightarrow y_1 = \frac{v_1^2}{2g} \overset{①}{\Rightarrow} y_1 = 0{,}8\,m$

Ισχύει $x = v_1 t = \sqrt{2 g y_1}\sqrt{\frac{2h_1}{g}}$ όπου $y_1 = H_1 - h_1$

$\Rightarrow x^2 = 2g(H_1 - h_1)\frac{2h_1}{g} \Rightarrow x^2 = 4H_1 h_1 - 4h_1^2 \Rightarrow 4h_1^2 - 4H_1 h_1 + x^2 = 0$

$\Delta = 16H_1^2 - 16x^2 \geq 0 \Rightarrow x \leq H_1 \rightsquigarrow x_{max} = H_1$ μέγιστη οριζόντια απόσταση

Άρα $h_1 = -\frac{-4H_1}{8} \Rightarrow h_1 = \frac{H_1}{2} \Rightarrow H_1 = 2h_1$ Όμως $y_1 + h_1 = H_1 \Rightarrow y_1 + h_1 = 2h_1$

$\Rightarrow h_1 = y_1 \Rightarrow \boxed{h_1 = 0{,}8\,m}$

Γ3 Ισχύει $H_1 = h_1 + y_1 = 1{,}6\,m$, $\Pi_{\varepsilon\rho} = \frac{V_1}{\Delta t} = \frac{H_1 A_{\delta o \chi}}{\Delta t} \Rightarrow \Delta t = \frac{H_1 A_{\delta o \chi}}{\Pi_{\varepsilon\rho}}$

$\Rightarrow \Delta t = \frac{1{,}6 \cdot 5 \cdot 10^{-3}}{2 \cdot 10^{-3}}$ sec $\Rightarrow \boxed{\Delta t = 4\,sec}$

Γ4 $\Pi'_{\varepsilon\rho} = 1{,}5\,\Pi_{\varepsilon\rho} = 1{,}5 \cdot 2 \cdot 10^{-3} = 3 \cdot 10^{-3}\,m^3/s$

Όταν η στάθμη σταθεροποιηθεί σε νέο ύψος H_1' τότε πάλι θα ισχύει $\Pi'_{\varepsilon\rho} = \Pi'_{o\pi\acute{\eta}\varsigma} = A v_1' \Rightarrow v_1' = \frac{\Pi'_{o\pi\acute{\eta}\varsigma}}{A} = \frac{3 \cdot 10^{-3}}{5 \cdot 10^{-4}}\,m/s \Rightarrow v_1' = 6\,m/s$.

Τότε η ① → $y_1' = \frac{{\upsilon_1'}^2}{2g} = \frac{36}{20}\,m \Rightarrow y_1' = 1,8m$

Άρα το νέο ύψος H_1' είναι: $H_1' = y_1' + h_1 = (1,8+0,8)m \Rightarrow H_1' = 2,6m$

Όμως $H = 2,5m < H_1' = 2,6m$ άρα το νερό ξεχυλίζει.

Γ5 Αφού η αρχική παροχή μειώνεται κατά 70%, η νέα παροχή θα είναι $\Pi_{\beta\rho}'' = 0,3\,\Pi_{\beta\rho} = 0,3\cdot 2\cdot 10^{-3} = 0,6\cdot 10^{-3}\,m^3/s$. Η στάθμη του νερού στο δοχείο κατεβαίνει με ταχύτητα $\upsilon = 0,2\,m/s$, οπότε ο όγκος του νερού που εισέρχεται στο δοχείο ($\Delta V_{\beta\rho}$) μαζί με τον όγκο που μειώνεται από το δοχείο ($|\Delta V_{\delta o\chi}|$) θα είναι ίσοι με τον όγκο του νερού που εξέρχεται από την οπή ($\Delta V_{o\pi\eta\varsigma}$) στον ίδιο χρόνο. Άρα $\Delta V_{\beta\rho} + |\Delta V_{\delta o\chi}| = \Delta V_{o\pi\eta\varsigma} \rightarrow$

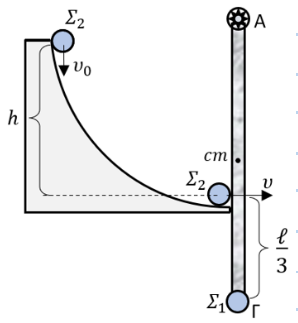
$$\frac{\Delta V_{\beta\rho}}{\Delta t} + \frac{|\Delta V_{\delta o\chi}|}{\Delta t} = \frac{\Delta V_{o\pi\eta\varsigma}}{\Delta t} \Rightarrow \Pi_{\beta\rho}'' + A_{\delta o\chi}\upsilon = A\cdot\upsilon' \Rightarrow 0,6\cdot 10^{-3} + 5\cdot 10^{-3}\cdot 0,2 = 5\cdot 10^{-4}\upsilon'$$

$$\Rightarrow 1,6\cdot 10^{-3} = 5\cdot 10^{-4}\upsilon' \Rightarrow \upsilon' = 3,2\,m/s.$$

Bernoulli από επιφάνεια στην οπή: $P_{atm} + \frac{1}{2}\rho\upsilon^2 + \rho g H_2 = P_{atm} + \frac{1}{2}\rho\upsilon'^2 + \rho g h_1$

$$H_2 = \frac{\upsilon'^2 - \upsilon^2}{2g} + h_1 = \left(\frac{3,2^2 - 0,2^2}{20} + 0,8\right)m = (0,51 + 0,8)\,m \Rightarrow \boxed{H_2 = 1,31m}$$

ΘΕΜΑ Δ



Δ1 Ισχύει $I_{o\lambda_A} = I_{\rho\alpha\beta_A} + I_{\Sigma_1}$

όπου $I_{\rho\alpha\beta_A} = I_{cm} + M(\ell/2)^2 = \frac{1}{12}M\ell^2 + M\frac{\ell^2}{4} \Rightarrow I_{\rho\alpha\beta_A} = \frac{1}{3}M\ell^2 = \frac{1}{3}kg\,m^2$

και $M = m_1 = m = 1kg$ $\quad I_{\Sigma_1} = m_1\ell^2 = 1\,kg\,m^2$

$$\Rightarrow I_{o\lambda_A} = \frac{1}{3}M\ell^2 + m_1\ell^2 = \frac{1}{3}m\ell^2 + m\ell^2 \Rightarrow \boxed{I_{o\lambda_A} = \frac{4}{3}m\ell^2 = \frac{4}{3}kg\,m^2}$$

Δ2 $\Sigma\tau_A = 0$, ΑΔΣ: $\vec{L}_{\pi\rho\iota\nu} = \vec{L}_{\mu\varepsilon\tau\dot\alpha} \Rightarrow \vec{L}_{\Sigma_1} = \vec{L}_{\sigma\upsilon\sigma\tau} \Rightarrow m_1\upsilon\frac{2\ell}{3} = I_{o\lambda}\,\omega$

$$\Rightarrow m\upsilon\frac{2\ell}{3} = \frac{4}{3}m\ell^2\omega \Rightarrow \upsilon = 2\ell\omega \Rightarrow \upsilon = 6m/s.$$

$K_{\pi\rho\iota\nu} = \frac{1}{2}m_1\upsilon^2 = \frac{1}{2}1\cdot 36\,J \Rightarrow K_{\pi\rho\iota\nu} = 18J$

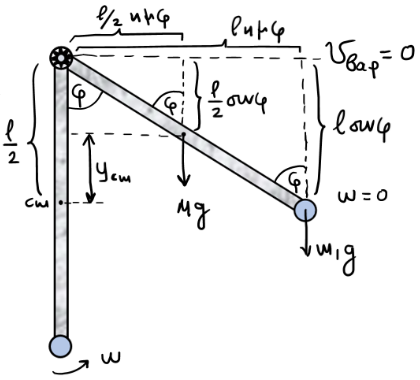
$K_{\mu\varepsilon\tau\dot\alpha} = K_{\rho\alpha\beta} + K_{\Sigma_1} = \frac{1}{2}I_{\rho\alpha\beta}\omega^2 + \frac{1}{2}I_{\Sigma_1}\omega^2 = \frac{1}{2}(I_{\rho\alpha\beta} + I_{\Sigma_1})\omega^2 = \frac{1}{2}I_{o\lambda}\,\omega^2$

$\Rightarrow K_{\mu\varepsilon\tau\dot\alpha} = \frac{1}{2}\frac{4}{3}\cdot 9\,J \Rightarrow \underline{K_{\mu\varepsilon\tau\dot\alpha} = 6J} < \underline{K_{\pi\rho\iota\nu} = 18J}$ <u>Ανελαστική κρούση</u>

Δ3 ΘΜΚΕ για Σ_2: $K_{2\tau\varepsilon\lambda} - K_{2\alpha\rho\chi} = W_{m_2 g} \Rightarrow \frac{1}{2}m_2\upsilon^2 - \frac{1}{2}m_2\upsilon_0^2 = m_2 g h$

$$\Rightarrow \upsilon_0^2 = \upsilon^2 - 2gh \Rightarrow \upsilon_0 = \sqrt{\upsilon^2 - 2gh} = \sqrt{36 - 11}\,m/s = \sqrt{25}\,m/s \Rightarrow \boxed{\upsilon_0 = 5\,m/s}$$

Δ4 $\frac{dL_{συστ}}{dt} = Στ_{εξ} = τ_{Mg} + τ_{m_1 g} \Rightarrow$

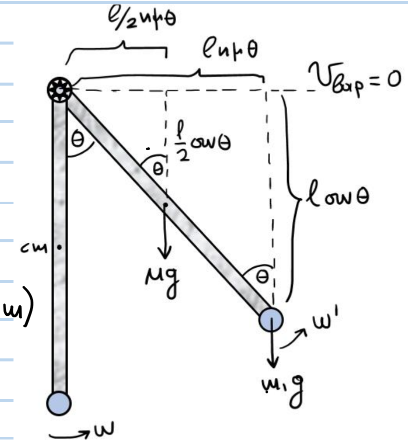


$$\frac{dL_{συστ}}{dt} = Mg\frac{\ell}{2}ημφ + m_1 g\ell ημφ = mg\frac{\ell}{2}ημφ + mg\ell ημφ$$

$$\Rightarrow \frac{dL_{συστ}}{dt} = \frac{3}{2}mg\ell ημφ = \frac{3}{2}10\cdot 1\cdot 0,8 \ N\cdot m$$

$$\Rightarrow \boxed{\frac{dL_{συστ}}{dt} = 12\ Nm, \otimes}$$

Δ5 α) Ο ρυθμός μεταβολής της κινητικής ενέργειας του συστήματος θα υπολογιστεί από τη σχέση



$$\frac{dK}{dt} = \frac{dW_{Στ}}{dt} = \frac{-Στ\cdot dθ}{dt} = -Στ\cdot ω'$$

όπου $Στ = τ_{Mg} + τ_{m_1 g} = Mg\frac{\ell}{2}ημθ + mg\ell ημθ \quad (M = m_1 = m)$

$$\Rightarrow Στ = \frac{3}{2}mg\ell ημθ = \frac{3}{2}1\cdot 10\cdot 1\cdot 0,6\ Nm$$

$$\Rightarrow Στ = 9\ Nm.$$

ΑΔΜΕ: $E_{MHX_{αρχ}} = E_{MHX_{τελ}} \Rightarrow K_{αρχ} + U_{M_{αρχ}} + U_{m_{αρχ}} = K_{τελ} + U_{M_{τελ}} + U_{m_{τελ}}$

$$\Rightarrow \frac{1}{2}I_{ολ}ω^2 - Mg\frac{\ell}{2} - m_1 g\ell = \frac{1}{2}I_{ολ}ω'^2 - Mg\frac{\ell}{2}συνθ - m_1 g\ell\, συνθ$$

$$\Rightarrow \frac{1}{2}I_{ολ}ω^2 - mg\frac{\ell}{2} - mg\ell = \frac{1}{2}I_{ολ}ω'^2 - 0,4mg\ell - 0,8mg\ell$$

$$\Rightarrow \frac{1}{2}I_{ολ}ω'^2 = \frac{1}{2}I_{ολ}ω^2 - 0,3mg\ell \Rightarrow \frac{1}{2}\frac{4}{3}ω'^2 = 6-3 \Rightarrow ω' = \frac{3\sqrt{2}}{2}\ rad/s$$

Άρα $\frac{dK}{dt} = -Στ\cdot ω' = -9\cdot\frac{3\sqrt{2}}{2} \Rightarrow \boxed{\frac{dK}{dt} = -\frac{27\sqrt{2}}{2}\ J/s}$

β) $\frac{dU_{βαρ(M)}}{dt} = -\frac{dW_{Mg}}{dt} = -\frac{dW_{τ_{Mg}}}{dt} = -\frac{-τ_{Mg}\,dθ}{dt} = τ_{Mg}\cdot ω'$

$$\Rightarrow \frac{dU_{βαρ(M)}}{dt} = +Mg\frac{\ell}{2}ημθ\cdot ω' = +1\cdot 10\frac{1}{2}\cdot 0,6\,\frac{3\sqrt{2}}{2} \Rightarrow \boxed{\frac{dU_{βαρ(M)}}{dt} = +4,5\sqrt{2}\ J/s}$$