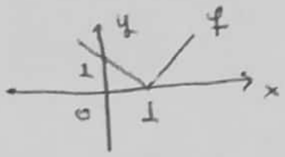
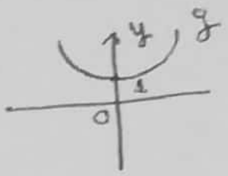
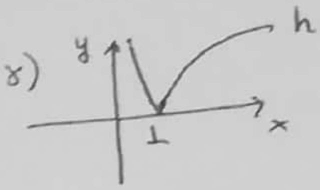
ΛΥΣΕΙΣ ΔΙΑΓΩΝΙΣΜΑΤΟΣ Β ΛΥΚΕΙΟΥ 3/4/2022

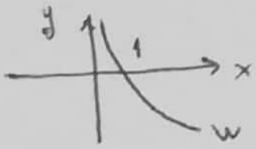
ΘΕΜΑ Α

A3

α) 

β) 

γ) 

δ) 

A4 α) Λ β) Σ γ) Σ δ) Σ ε) Σ

ΘΕΜΑ Β

B1 α) $\lim\limits_{x\to 1^-} \frac{x^3-2x+1}{x-1} = \lim\limits_{x\to 1^-} \frac{\cancel{(x-1)}(x^2+x-1)}{\cancel{x-1}} = \boxed{1}$

1	0	−2	1	1
↓	1	1	−1	
1	1	−1	0	

$$\lim_{x\to 1^+} \frac{2\sqrt{x^2+3}-4}{x-1} = \lim_{x\to 1^+} \frac{(2\sqrt{x^2+3}-4)(2\sqrt{x^2+3}+4)}{(x-1)(2\sqrt{x^2+3}+4)} =$$

$$= \lim_{x\to 1^+} \frac{4(x^2+3)-16}{(x-1)(2\sqrt{x^2+3}+4)} = \lim_{x\to 1^+} \frac{4(x^2+3-4)}{(x-1)\cdot(2\sqrt{x^2+3}+4)} = \lim_{x\to 1^+} \frac{4\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(2\sqrt{x^2+3}+4)}$$

$$= \frac{4\cdot 2}{2\cdot 2+4} = \frac{8}{8} = \boxed{1} \quad \text{άρα } \lim_{x\to 1} f(x) = \boxed{1}$$

• $\lim\limits_{x\to 0} \frac{x^3-2x+1}{x-1} = \frac{1}{-1} = \boxed{-1}$

β) $y^2+3y-4 = (y-1)(y+4)$

$\Delta = 9+16 = 25$

$y = \frac{-3+5}{2} = 1$, $\quad y = \frac{-3-5}{2} = \frac{-8}{2} = -4$

$$\lim_{x\to 1} \frac{\cancel{(f(x)-1)}(f(x)+4)}{\cancel{(f(x)-1)}(f(x)+1)} = \boxed{\frac{5}{2}}$$

$\boxed{B2}$ α) $A_f = [-4,-1) \cup (-1,5)$

$f(A) = [-2,3)$

β) i) $\lim\limits_{x\to -1^-} f(x) = 1 \neq \lim\limits_{x\to -1^+} f(x) = 3 \quad \nexists$

ii) $\lim\limits_{x\to 3^-} f(x) = 1 \neq \lim\limits_{x\to 3^+} f(x) = 3 \quad \nexists$

iii) $\lim\limits_{x\to -4} f(x) = 1$

iv) $\lim\limits_{x\to 1} f(x) = -2$

γ) i) $f(x) = 2 \longrightarrow 3$ # ΡΙΖΕΣ

ii) $f(x) = 1 \longrightarrow 5$ # ΡΙΖΕΣ

iii) $f(x) = -\sqrt{2} \longrightarrow 4$ # ΡΙΖΕΣ

ΘΕΜΑ Γ

$\boxed{Γ_1}$ $f(1) = 3 \Leftrightarrow \dfrac{1+10+2\alpha}{1+\alpha} = 3 \Leftrightarrow 11 + 2\alpha = 3 + 3\alpha \Leftrightarrow$

$\Leftrightarrow \boxed{\alpha = 8}$ Για $f(x) = \dfrac{x^2+10x+16}{x^3+8}$

$\boxed{Γ_2}$ $x^3+8 \neq 0 \Rightarrow x^3 \neq -8 \Rightarrow x \neq -2$ οπότε $\boxed{A_f = \mathbb{R} - \{-2\}}$

- $x^3 + 8 = (x+2)(x^2-2x+4)$
- $x^2+10x+16 = (x+2)(x+8)$

$\Delta = 100 - 64 = 36$

$x_{1,2} = \begin{cases} \dfrac{-10+6}{2} = \dfrac{-4}{2} = -2 \\ \dfrac{-10-6}{2} = \dfrac{-16}{2} = -8 \end{cases}$

$\lim\limits_{x\to -2} f(x) = \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x+8)}{\cancel{(x+2)}(x^2-2x+4)}$

$= \lim\limits_{x\to -2} \dfrac{x+8}{x^2-2x+4} = \dfrac{-2+8}{4+4+4} = \dfrac{6}{12} = \boxed{\dfrac{1}{2}}$

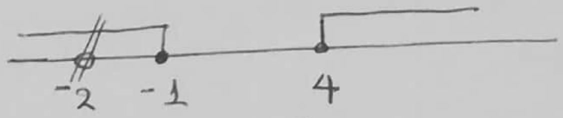
άρα $f(x)=\dfrac{x+8}{x^2-2x+4}\ ,\ x\neq -2$

$\boxed{\Gamma_3}$

$f(x)\le 1 \iff \dfrac{x+8}{x^2-2x+4}\le 1 \underset{\substack{x^2-2x+4>0\\ \Delta<0}}{\iff} x+8\le x^2-2x+4$

$\iff x^2-3x-4\ge 0 \qquad \Delta=9+16=25 \qquad x_{1,2}=\begin{cases}\frac{3+5}{2}=\frac{8}{2}=4\\[4pt] \frac{3-5}{2}=\frac{-2}{2}=-1\end{cases}$

	-1		4	
$+$	0	$-$	0	$+$

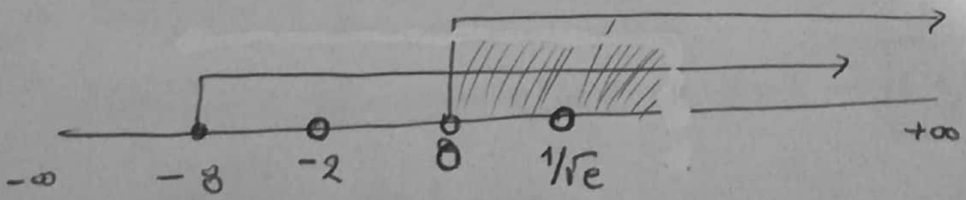


άρα $\boxed{x\in(-\infty,-2)\cup(-2,-1]\cup[4,+\infty)}$

$\boxed{\Gamma_4}$

Πρέπει: $\begin{cases}\bullet\ f(x)\ge 0 \to \boxed{x\ge -8}\\ \bullet\ \boxed{x\neq -2}\\ \bullet\ 2\ln x+1\neq 0 \to \boxed{x\neq 1/\sqrt{e}}\\ \bullet\ \boxed{x>0}\end{cases}$

- $f(x)\ge 0 \iff \dfrac{x+8}{x^2-2x+4}\ge 0 \underset{x\neq -2}{\iff} x+8\ge 0 \iff x\ge -8$
- $\left(-2^x+4>0 \iff -2^x>-4 \iff 2^x<4 \iff x<2\right)$
- $2\ln x+1=0 \iff \ln x=-1/2 \iff x=e^{-\frac{1}{2}} \iff x=\dfrac{1}{\sqrt{e}}$



άρα $\boxed{A_g=\left(0,\frac{1}{\sqrt{e}}\right)\cup\left(\frac{1}{\sqrt{e}},+\infty\right)}$

$\boxed{\Gamma_5}$

x	$f(-2023)$	-8		-2	$f(2022)$
$f(x)$	$-$	0	$+$	$\Vert$	$+$

Από τον πίνακα προσήμου της f έχουμε:
$f(2022)>0$ κ' $f(-2023)<0$

• $e^{x^2}-1 \geq 0 \Leftrightarrow e^{x^2} \geq 1 \Leftrightarrow x^2 \geq 0$ ΙΣΧΥΕΙ $\forall x \in \mathbb{R}$

ΚΑΙ ΙΣΟΝ ΙΣΧΥΕΙ ΜΟΝΟ ΟΤΑΝ $x=0$

• $1>0 \Leftrightarrow x^2+1>x^2 \Leftrightarrow \sqrt{x^2+1} > \sqrt{x^2} \Leftrightarrow \sqrt{x^2+1} > |x| \geq x$

$\Leftrightarrow \sqrt{x^2+1} > x \Leftrightarrow \sqrt{x^2+1}-x>0, \ \forall x \in \mathbb{R}$

Αρα $\forall x \in \mathbb{R}^*$ $\dfrac{f(2022)\cdot(e^{x^2}-1)}{f(-2023)\cdot(\sqrt{x^2+1}-x)} < 0$ ΚΑΙ ΓΙΑ $x=0$ ΙΣΧΥΕΙ Η ΙΣΟΤΗΤΑ

ΘΕΜΑ Δ

$\boxed{Δ1}$ $A_f = A_g = \mathbb{R}$

▸ $f(x) = g(x) \Leftrightarrow \dfrac{2e^{2x}+1}{\cancel{e^x+2}} = \dfrac{e^{x+1}+e^{2x}+e^x-e+1}{\cancel{e^x+2}}$

$\Leftrightarrow 2e^{2x}+\cancel{1} = e^x \cdot e + e^{2x} + e^x - e + \cancel{1}$

$\Leftrightarrow e^{2x} = (e+1)e^x - e \Leftrightarrow e^{2x}-(e+1)e^x+e=0$

$\Delta = (e+1)^2-4e = e^2+2e+1-4e = e^2-2e+1 = (e-1)^2$

$e^x = \begin{cases} \dfrac{\cancel{e}+1-\cancel{e}+1}{2} = \dfrac{2}{2} = 1 \rightarrow e^x = 1 \rightarrow x=0 \rightarrow \boxed{(0,1)} \\ \dfrac{e+\cancel{1}+e-\cancel{1}}{2} = \dfrac{2e}{2} = e \rightarrow e^x = e \rightarrow x=1 \rightarrow \boxed{\left(1, \dfrac{2e^2+1}{e+2}\right)} \end{cases}$

$\boxed{Δ2}$ $y = e^{\ln\frac{f(1)}{g(1)}} = e^{\ln 1} = e^0 = 1$

▸ $f(x) > 1 \Leftrightarrow \dfrac{2e^{2x}+1}{e^x+2} > 1 \Leftrightarrow 2e^{2x}+1 > e^x+2$

$\Leftrightarrow 2e^{2x}-e^x-1>0$

$\Delta = 1+8=9$

$e^x = \begin{cases} \dfrac{1+3}{4} = 1 \\ \dfrac{1-3}{4} = \dfrac{-2}{4} = -\dfrac{1}{2} \end{cases}$

	$-1/2$		1	
$+$	ϕ	$-$	ϕ	$+$

$e^x > -\dfrac{1}{2}$ ή $e^x > 1 \Leftrightarrow \boxed{x>0}$

ΙΣΧΥΕΙ $\forall x \in \mathbb{R}$

$\boxed{\Delta 3}$ $f(0)=1$ άρα $\alpha^x < 1$

- Αν $\underline{\alpha \in (0,1)}$: $\alpha^x < 1 \Leftrightarrow \alpha^x < \alpha^0 \overset{\alpha^x \downarrow}{\Leftrightarrow} \boxed{x > 0}$
- Αν $\underline{\alpha \in (1,+\infty)}$: $\alpha^x < 1 \Leftrightarrow \alpha^x < \alpha^0 \overset{\alpha^x \uparrow}{\Leftrightarrow} \boxed{x < 0}$

$\boxed{\Delta 4}$ $f(1) = g(1)$

$$(x^3+x^2-x-1)^{2022} = \sigma\upsilon\nu\left(\frac{\pi x-\pi}{4}\right) - 1$$

<u>Είναι:</u>

$\sigma\upsilon\nu\left(\frac{\pi x-\pi}{4}\right) - 1 \leq 0 \Leftrightarrow (x^3+x^2-x-1)^{2022} \leq 0$

Όμως $(x^3+x^2-x-1)^{2022} \geq 0 \quad \forall x \in \mathbb{R}$

Άρα $x^3+x^2-x-1=0$

1	1	−1	−1	1
↓	1	2	1	
1	2	1	0	

άρα $(x-1)(x^2+2x+1)=0$

$(x-1)(x+1)^2=0 \Leftrightarrow x=1$ ή $x=-1$

<u>Επαλήθευση:</u>

Για $\boxed{x=1}$: $(\cancel{1}+\cancel{1}-\cancel{1}-\cancel{1})^{2022} = \sigma\upsilon\nu\left(\frac{\pi-\pi}{4}\right)-1$

$0 = \sigma\upsilon\nu 0 - 1 \Leftrightarrow 0=0$ <u>Ισχύει</u>

Για $\boxed{x=-1}$: $(-\cancel{1}+\cancel{1}+\cancel{1}-\cancel{1})^{2022} = \sigma\upsilon\nu\left(\frac{-\pi-\pi}{4}\right)-1$

$0 = \sigma\upsilon\nu\left(-\frac{2\pi}{4}\right)-1 \Leftrightarrow 0 = \sigma\upsilon\nu\left(-\frac{\pi}{2}\right)-1$

$0 = \sigma\upsilon\nu\frac{\pi}{2}-1 \Leftrightarrow 0=-1$ Άτοπο

Άρα $\boxed{x=1}$

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