

## Θέμα Α

A<sub>1</sub>. Σχολικό βιβλίο ΓΕΑ. 63

A<sub>2</sub>. iii

A<sub>3</sub>.

α. Λάθος

β. Σωστό

γ. Λάθος

δ. Λάθος

ε. Λάθος

A<sub>4</sub>.

α.  $A \cdot B = (4 - \sqrt{15})(4 + \sqrt{15}) = 4^2 - \sqrt{15}^2 = 16 - 15 = 1$

β.  $\Gamma = A^2 + B^2 = (4 - \sqrt{15})^2 + (4 + \sqrt{15})^2 = 16 - 8\sqrt{15} + 15 + 16 + 8\sqrt{15} + 15 = 62$

## Θέμα Β

1.

α.  $1 < x < 2$

και  $1 < x < 2$

$$1 - 1 < x - 1 < 2 - 1$$

$$1 - 2 < x - 2 < 2 - 2$$

$$0 < x - 1 < 1$$

$$-1 < x - 2 < 0$$

αρα  $A = x - 1 + x - 2 = 2x - 3$

β.  $x < 1$

και  $x < 1$

$$x - 1 < 1 - 1$$

$$x - 2 < 1 - 2$$

$$x - 1 < 0$$

$$x - 2 < -1$$

αρα  $A = -x + 1 + x - 2 = -1$

①

B2.

$$\begin{aligned} \alpha. A+B &= \frac{1}{3+\sqrt{3}} + \frac{1}{3-\sqrt{3}} = \frac{3-\sqrt{3}}{(3+\sqrt{3})(3-\sqrt{3})} + \frac{3+\sqrt{3}}{(3+\sqrt{3})(3-\sqrt{3})} = \\ &= \frac{3-\sqrt{3}+3+\sqrt{3}}{3^2-\sqrt{3}^2} = \frac{6}{9-3} = 1 \end{aligned}$$

$$\beta. A \cdot B = \frac{1}{3+\sqrt{3}} \cdot \frac{1}{3-\sqrt{3}} = \frac{1}{(3+\sqrt{3})(3-\sqrt{3})} = \frac{1}{3^2-\sqrt{3}^2} = \frac{1}{9-3} = \frac{1}{6}$$

Θέμα Γ

Γ1.

$$\begin{aligned} \alpha. (2+3\sqrt{5})^2 &= 4+12\sqrt{5}+45=49+12\sqrt{5} \\ (2-3\sqrt{5})^2 &= 4-12\sqrt{5}+45=49-12\sqrt{5} \end{aligned}$$

$$\begin{aligned} \beta. \sqrt{49-12\sqrt{5}} - \sqrt{49+12\sqrt{5}} &\stackrel{\alpha}{=} \sqrt{(2-3\sqrt{5})^2} - \sqrt{(2+3\sqrt{5})^2} = \\ &= |2-3\sqrt{5}| - |2+3\sqrt{5}| = -2+3\sqrt{5} - 2-3\sqrt{5} = -4 \end{aligned}$$

Γ2.

$$\begin{aligned} \sqrt[8]{3\sqrt{3}\sqrt[5]{3}} &= \sqrt[8]{3\sqrt{3}\cdot 3^{\frac{1}{5}}} = \sqrt[8]{3\sqrt{3^{\frac{6}{5}}}} = \sqrt[8]{3\cdot 3^{\frac{6}{10}}} = \sqrt[8]{3^{\frac{16}{10}}} = \sqrt[8]{3^{\frac{16}{80}}} = 3^{\frac{16}{80}} = \\ &= 3^{\frac{2}{10}} = \sqrt[5]{3} \end{aligned}$$

Γ3.

$$\begin{aligned} ||x|-2|-3|=1 &\Leftrightarrow |x|-2-3=1 \quad \vee \quad |x|-2-3=-1 \\ &|x|-2=4 \quad \vee \quad |x|-2=2 \\ |x|-2=4 \quad \vee \quad |x|-2=-4 \quad \vee \quad |x|-2=2 \quad \vee \quad |x|-2=-2 \\ |x|=6 \quad \vee \quad |x|=-2 \quad \vee \quad |x|=4 \quad \vee \quad |x|=0 \\ x=6 \quad \vee \quad x=-6 \quad \text{αδύνατα} \quad \vee \quad x=4 \quad \vee \quad x=-4 \quad \vee \quad x=0 \end{aligned}$$

(2)

## Όερα Δ

Δ<sub>1</sub>.

$$\begin{aligned}x \cdot y \cdot z &= \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2-\sqrt{2+\sqrt{3}}} \cdot \sqrt{2+\sqrt{3}} = \\&= \sqrt{(2+\sqrt{2+\sqrt{3}})(2-\sqrt{2+\sqrt{3}})(2+\sqrt{3})} = \\&= \sqrt{(2^2 - \sqrt{2+\sqrt{3}}^2)(2+\sqrt{3})} = \\&= \sqrt{(4-2-\sqrt{3})(2+\sqrt{3})} = \\&= \sqrt{(2-\sqrt{3})(2+\sqrt{3})} = \sqrt{2^2 - \sqrt{3}^2} = \sqrt{4-3} = \sqrt{1} = 1\end{aligned}$$

Δ<sub>2</sub>.

$$\begin{aligned}\frac{2|2x-1|-3}{2} + \frac{|2x-1|+1}{4} &= |2x-1| \Leftrightarrow \\4 \cdot \frac{2|2x-1|-3}{2} + 4 \cdot \frac{|2x-1|+1}{4} &= 4|2x-1| \Leftrightarrow \\2(2|2x-1|-3) + |2x-1| + 1 &= 4|2x-1| \Leftrightarrow \\4|2x-1| - 6 + |2x-1| + 1 &= 4|2x-1| \Leftrightarrow \\|2x-1| = 5 &\Leftrightarrow 2x-1=5 \quad \eta \quad 2x-1=-5 \\2x &= 6 \quad \eta \quad 2x = -4 \\x &= 3 \quad \quad \quad x = -2\end{aligned}$$

Δ<sub>3</sub>.

$$\begin{aligned}\frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}} - \frac{x}{2} &= \frac{(\sqrt{x+2} + \sqrt{x-2})(\sqrt{x+2} + \sqrt{x-2})}{(\sqrt{x+2} - \sqrt{x-2})(\sqrt{x+2} + \sqrt{x-2})} - \frac{x}{2} = \\&= \frac{(\sqrt{x+2} + \sqrt{x-2})^2}{\sqrt{x+2}^2 - \sqrt{x-2}^2} - \frac{x}{2} \stackrel{\substack{x+2 \geq 0 \\ x-2 \geq 0}}{=} \frac{x+2+2\sqrt{x+2}\sqrt{x-2}+x-2}{x+2-x+2} - \frac{x}{2} = \\&= \frac{2x+2\sqrt{(x+2)(x-2)}}{4} - \frac{x}{2} = \frac{2x}{4} + \frac{\sqrt{x^2-4}}{2} - \frac{x}{2} = \frac{1}{2}\sqrt{x^2-4}\end{aligned}$$

$$\Delta_4, \quad \sqrt{a+3-\sqrt{12a}} = \sqrt{a+3-2\sqrt{3a}} \stackrel{a \geq 0}{=} \sqrt{a^2 - 2\sqrt{3}a + \sqrt{3}^2} = \sqrt{(\sqrt{a} - \sqrt{3})^2} =$$

$$= |\sqrt{a} - \sqrt{3}| = \begin{cases} \sqrt{a} - \sqrt{3}, & \text{or } a \geq 3 \\ 0, & \text{or } a = 3 \\ -\sqrt{a} + \sqrt{3}, & \text{or } 0 \leq a < 3 \end{cases}$$