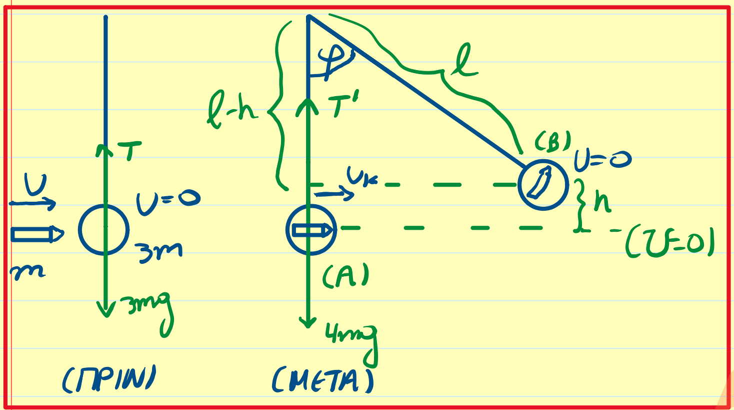
ΘΕΜΑ Α

Α1 β **Α2** γ **Α3** α **Α4** δ

Α5 α) Λ β) Σ γ) Λ δ) Σ ε) Σ

ΘΕΜΑ Β

Β1 Σωστή απάντηση (β)



ΠΡΙΝ: $\Sigma F_y = 0 \Rightarrow T = 3mg$

ΜΕΤΑ: $\Sigma F_R = 4m\cdot\frac{\upsilon_\kappa^2}{\ell}$

$\Rightarrow T' - 4mg = 4m\cdot\frac{\upsilon_\kappa^2}{\ell}$

$\Rightarrow T' = 4mg + 4m\cdot\frac{\upsilon_\kappa^2}{\ell}$

• $T' = 2\cdot T \Rightarrow 4mg + 4m\cdot\frac{\upsilon_\kappa^2}{\ell} = 6mg \Rightarrow 4\cancel{m}\cdot\frac{\upsilon_\kappa^2}{\ell} = 2\cancel{m}g$

$\Rightarrow \upsilon_\kappa^2 = \frac{g\ell}{2} \Rightarrow \upsilon_\kappa = \sqrt{\frac{g\ell}{2}}$

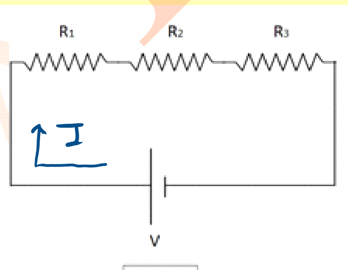
• Α.Δ.Μ.Ε.(Α→Β)

$K_A + \cancel{U_A}^{0} = \cancel{K_B}^{0} + U_B \Rightarrow \frac{1}{2}\cdot 4\cancel{m}\cdot\upsilon_\kappa^2 = 4\cancel{m}g\cdot h \Rightarrow \frac{1}{2}\cdot\frac{\cancel{g}\ell}{2} = \cancel{g}\cdot h$

$\Rightarrow h = \frac{\ell}{4}$

Άρα: $\sigma\upsilon\nu\varphi = \frac{\ell - h}{\ell} = \frac{\ell - \frac{\ell}{4}}{\ell} = \frac{\frac{3\ell}{4}}{\ell} \Rightarrow \boxed{\sigma\upsilon\nu\varphi = \frac{3}{4}}$

Β2 Σωστή απάντηση (γ)

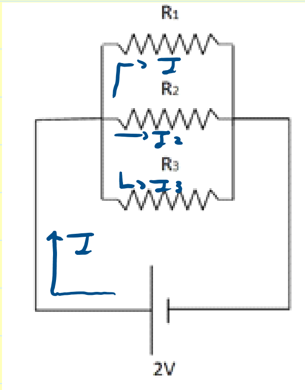


Σχήμα 1

$R_{o\lambda} = R_1 + R_2 + R_3 = 3\cdot R$

$I = \frac{V}{R_{o\lambda}} = \frac{V}{3R}$

$P_1 = I^2\cdot R_1 = \left(\frac{V}{3R}\right)^2\cdot R = \frac{V^2}{9R^2}\cdot R \Rightarrow P_1 = \frac{V^2}{9R}$



Σχήμα 2

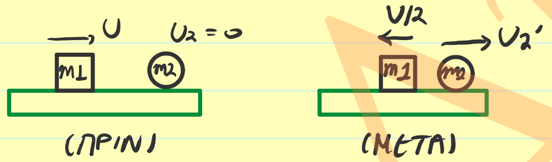
$$V_1 = V_2 = V_3 = 2V$$

$$P_1' = \frac{V_1^2}{R_1} = \frac{(2V)^2}{R} = \frac{4V^2}{R}$$

$$\boxed{\begin{aligned} &\text{ή}\quad \frac{1}{R_{ολ}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \Rightarrow R_{ολ} = \frac{R}{3} \\ &I = \frac{2V}{R_{ολ}} \Rightarrow I = \frac{2V}{R/3} = \frac{6V}{R} \\ &\text{Όμως: } V_1 = V_2 = V_3 \Rightarrow I_1 \cdot \cancel{R_1} = I_2 \cdot \cancel{R_2} = I_3 \cdot \cancel{R_3} \\ &\Rightarrow I_1 = I_2 = I_3 = \frac{I}{3} \\ &P_1' = I_1^2 \cdot R_1 = \frac{I^2}{9} \cdot R = \frac{36V^2}{9R^2} \cdot \cancel{R} \Rightarrow P_1' = \frac{4V^2}{R} \end{aligned}}$$

Άρα: $\frac{P_1}{P_1'} = \frac{\frac{V^2}{9R}}{\frac{4V^2}{R}} \Rightarrow \boxed{\frac{P_1}{P_1'} = \frac{1}{36}}$

B3 i) Σωστή απάντηση (β)



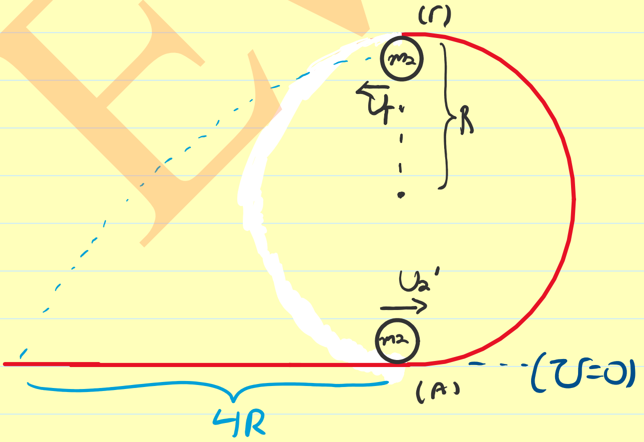
Α.Δ.Ο.

$$\vec{p}_1 + \cancel{\vec{p}_2}^{\,0} = \vec{p}_1\,' + \vec{p}_2\,'$$

$$\overset{(+)}{\Rightarrow} m \cdot υ = -m \cdot \frac{υ}{2} + 3m \cdot υ_2'$$

$$\Rightarrow \frac{\cancel{3}}{2} \cancel{m}\, υ = \cancel{3}\cancel{m} \cdot υ_2' \Rightarrow \boxed{υ_2' = \frac{υ}{2}}$$

ii) Σωστή απάντηση (α)



Οριζόντια βολή:

$$x = υ_Γ \cdot t_{ολ} \Rightarrow 4R = υ_Γ \sqrt{\frac{2 \cdot 2R}{g}}$$

$$\Rightarrow υ_Γ = \sqrt{\frac{16 g R^2}{4R}} \Rightarrow υ_Γ = \sqrt{4gR}$$

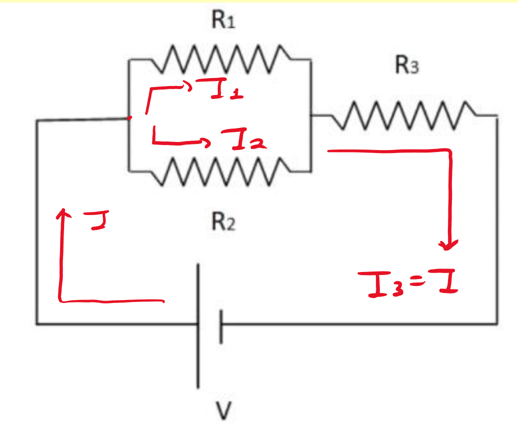
Α.Δ.Μ.Ε. (Α→Γ)

$K_A + U_A = K_\Gamma + U_\Gamma \Rightarrow {}^{2}\frac{1}{2}\cancel{m_2}\,u_2'^2 = {}^{2}\frac{1}{2}\cancel{m_2}\,u_\Gamma^2 + {}^{2}\cancel{m_2}\,g\cdot 2R$

$\Rightarrow u_2'^2 = (\sqrt{4gR})^2 + 4gR \Rightarrow u_2' = \sqrt{8gR} = 2\cdot\sqrt{2gR}$

Άρα: $u_2' = \frac{u}{2} \Rightarrow u = 2\cdot u_2' \Rightarrow u = 4\cdot\sqrt{2gR}$

$\boxed{\text{ΘΕΜΑ Γ}}$

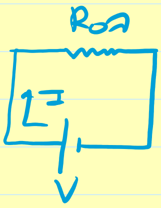


$\boxed{\Gamma 1}$ $R_{ολ} = ;$

$$R_{1,2} = \frac{R_1\cdot R_2}{R_1+R_2} = \frac{3\cdot 6}{3+6} = 2\Omega$$

$$R_{ολ} = R_{1,2} + R_3 \Rightarrow \boxed{R_{ολ} = 6\Omega}$$

$\boxed{\Gamma 2}$ $V_2 = ;$



$$I = \frac{V}{R_{ολ}} = \frac{36}{6} \Rightarrow I = 6A$$

• $V_1 = V_2 \Rightarrow I_1\cdot R_1 = I_2\cdot R_2 \Rightarrow I_1\cdot 3 = I_2\cdot 6$

$\Rightarrow I_1 = 2I_2$

Όμως: $I_1 + I_2 = I \Rightarrow 2\cdot I_2 + I_2 = 6 \Rightarrow 3I_2 = 6 \Rightarrow I_2 = 2A$

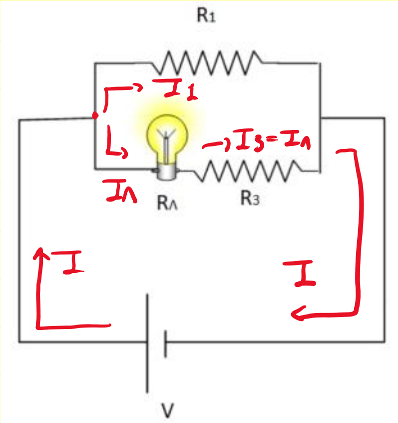
Άρα: $V_2 = I_2\cdot R_2 \Rightarrow \boxed{V_2 = 12V}$

$\boxed{\Gamma 3}$ $Q_{R_1} = ;$, $\Delta t = 5min = 300s$

$I_1 = 2\cdot I_2 \Rightarrow I_1 = 4A$

$Q_{R_1} = I_1^2\cdot R_1\cdot \Delta t = 4^2\cdot 3\cdot 300 \Rightarrow \boxed{Q_{R_1} = 14.400J}$

$\boxed{\Gamma 4}$ Κανονική λειτουργία;



Για τη λάμπα:

$$P_\kappa = \frac{V_\kappa^2}{R_\lambda} \Rightarrow R_\lambda = \frac{V_\kappa^2}{P_\kappa} = \frac{36}{18} \Rightarrow R_\lambda = 2\Omega$$

$$I_\kappa = \frac{P_\kappa}{V_\kappa} = \frac{18}{6} \Rightarrow I_\kappa = 3A$$

$$R_{\lambda,3} = R_\lambda + R_3 = 6\Omega$$

$$R_{o\lambda} = \frac{R_{\lambda,3}\cdot R_1}{R_{\lambda,3}+R_1} = \frac{6\cdot 3}{6+3} = 2\Omega$$

$$I = \frac{V}{R_{o\lambda}} = \frac{36}{2} = 18A$$

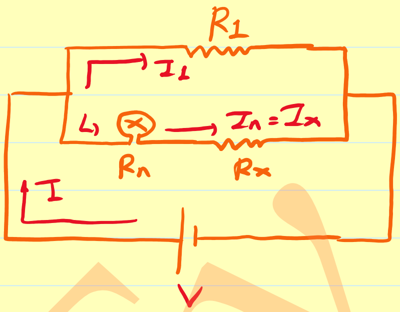
Όμως: $V_1 = V_{\lambda,3} \Rightarrow I_1\cdot R_1 = I_\lambda\cdot R_{\lambda,3} \Rightarrow I_1\cdot 3 = I_\lambda 6$

$\Rightarrow I_1 = 2I_\lambda$

$I_1 + I_\lambda = I \Rightarrow 2I_\lambda + I_\lambda = I \Rightarrow 3\cdot I_\lambda = 18 \Rightarrow I_\lambda = 6A$

Άρα: $I_\lambda = 6A > I_\kappa = 3A$ → Υπερλειτουργεί

Γ5 $R_x = ;$



Άρα λειτουργεί κανονικά:

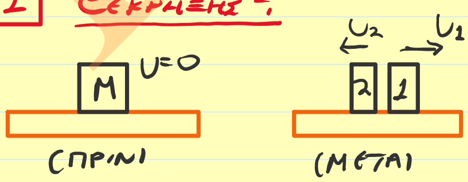
$I_\lambda = I_\kappa = 3A$ και $V_\lambda = V_\kappa = 6V$

Όμως: $V_x + V_\lambda = V \Rightarrow V_x = 36 - 6 = 30V$

$$I_x = \frac{V_x}{R_x} \Rightarrow R_x = \frac{V_x}{I_x} = \frac{30}{3} \Rightarrow \boxed{R_x = 10\Omega}$$

ΘΕΜΑ Δ

Δ1 Εκρηξης;



Α.Δ.Ο.

$$\vec{p}_{o\lambda,\pi\rho\iota\nu} = \vec{p}_{o\lambda,\mu\varepsilon\tau\acute{\alpha}} \overset{(+)}{\Rightarrow} 0 = m_1 U_1 - m_2 U_2$$

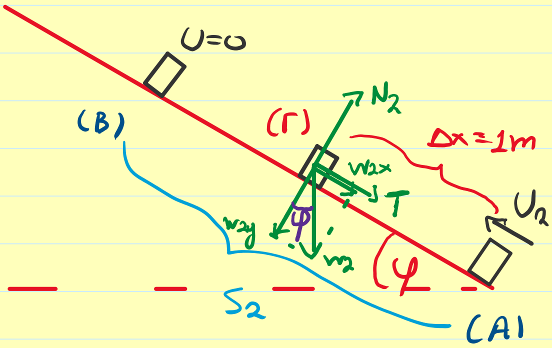
$\Rightarrow 2\cdot 9 = 6\cdot U_2 \Rightarrow U_2 = 3\ m/s$

$K_{o\lambda,\pi\rho\iota\nu} = 0$

$$K_{ολ,μετά} = \frac{1}{2} m_1 υ_1^2 + \frac{1}{2} m_2 υ_2^2 = \frac{1}{2} 2\cdot 9^2 + \frac{1}{2} 6\cdot 3^2 = 81 + 27 = 108J$$

$$E_{εκρ.} = K_{ολ,μετά} - K_{ολ,πριν} \Rightarrow \boxed{E_{εκρ.} = 108J}$$

$\boxed{Δ2}$ $S_2 = ;$



$$w_{2x} = w_2 \cdot ημφ = 30N$$
$$w_{2y} = w_2 \cdot συνφ = 30\sqrt{3}N$$
$$ΣF_y = 0 \Rightarrow N = w_{2y} = 30\sqrt{3}N$$
$$T = μ\cdot N = \frac{\sqrt{3}}{3} 30\sqrt{3} \Rightarrow T = 30N$$

Θ.Μ.Κ.Ε.(Α→Β)

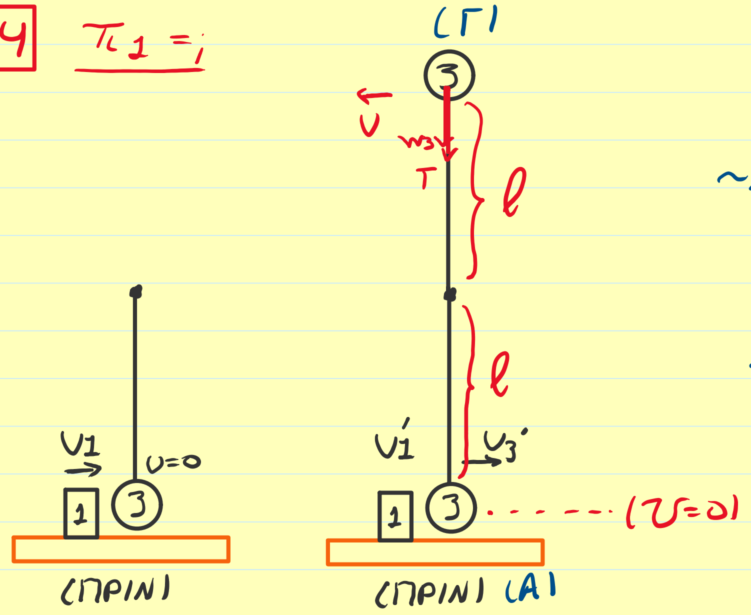
$$\cancel{K_{τελ}}^{0} - K_{αρχ} = \cancel{W_{N_2}}^{0} + \cancel{W_{w_{2y}}}^{0} + W_T + W_{w_{2x}}$$
$$\Rightarrow -\frac{1}{2} m_2 υ_2^2 = -T\cdot S_2 - w_{2x}\cdot S_2$$
$$\Rightarrow -\frac{1}{2}\cdot 6\cdot 3^2 = -30\cdot S_2 - 30\cdot S_2$$
$$\Rightarrow -27 = -60 S_2 \Rightarrow \boxed{S_2 = 0{,}45m}$$

$\boxed{Δ3}$ $Δx = 0{,}25m$, $\frac{dK_2}{dt} = ;$

Θ.Μ.Κ.Ε.(Α→Γ)

$$\frac{1}{2} m_2 υ^2 - \frac{1}{2} m_2 υ_2^2 = -T\cdot Δx - w_{2x}\cdot Δx$$
$$\Rightarrow {}^{2}\frac{1}{2}\cdot 6\cdot υ^2 - {}^{2}\frac{1}{2}\cdot 6\cdot 3^2 = {}^{2}-30\cdot Δx \;{}^{2}-30\cdot Δx$$
$$\Rightarrow υ^2 - 9 = -\frac{120}{6}\cdot 0{,}25 \Rightarrow υ^2 = 4 \Rightarrow υ = 2 m/s$$
$$\frac{dK_2}{dt} = ΣF_x \cdot υ = -(w_{2x} + T)\cdot υ \Rightarrow \boxed{\frac{dK_2}{dt} = -120 J/s}$$

$\boxed{\Delta 4}$ $\pi_1 = ;$



Το σώμα μάζας m_3 εκτελεί οριακή ανακύκλωση.

~> Στην ανώτερη θέση:

$$T + m_3 g = m_3 \frac{U^2}{\ell} \Rightarrow T = m_3 \frac{U^2}{\ell} - m_3 g$$

$$\cdot\ T \geq 0 \Rightarrow m_3 \frac{U^2}{\ell} \geq m_3 g$$

$$\Rightarrow U = \sqrt{g\ell}$$

~> A.Δ.Μ.Ε. (Α→Γ)

$$K_A + \cancel{U_A}^{\,0} = K_\Gamma + U_\Gamma \Rightarrow \frac{1}{2}\cancel{m_3} U_3'^2 = \frac{1}{2}\cancel{m_3}(\sqrt{g\ell})^2 + \cancel{m_3} g\, 2\ell$$

$$\Rightarrow U_3'^2 = g\ell + 4 g\ell \Rightarrow U_3' = \sqrt{5 g\ell} = 5\,m/s$$

A.Δ.Ο.

$$\vec{p}_1 + \vec{p}_3 = \vec{p}_1{}' + \vec{p}_3{}' \overset{(+)}{\Rightarrow} m_1 U_1 + 0 = m_1 U_1' + m_3 U_3'$$

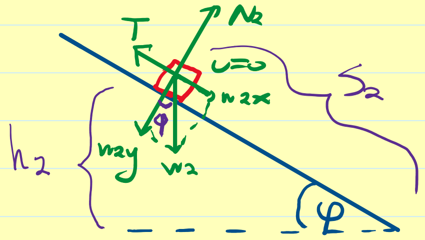
$$\Rightarrow 2 \cdot 9 = 2 \cdot U_1' + 4 \cdot 5 \Rightarrow 18 - 20 = 2 \cdot U_1' \Rightarrow U_1' = -1\,m/s$$

$$|U_1'| = 1\,m/s \quad (\leftarrow)$$

$$\left.\begin{aligned} K_1 &= \frac{1}{2} m_1 U_1^2 = \frac{1}{2} \cdot 2 \cdot 9^2 = 81\,J \\ K_1' &= \frac{1}{2} m_1 U_1'^2 = \frac{1}{2} \cdot 2 \cdot 1^2 = 1\,J \end{aligned}\right\} \Rightarrow \Delta K_1 = 1 - 81 = -80\,J$$

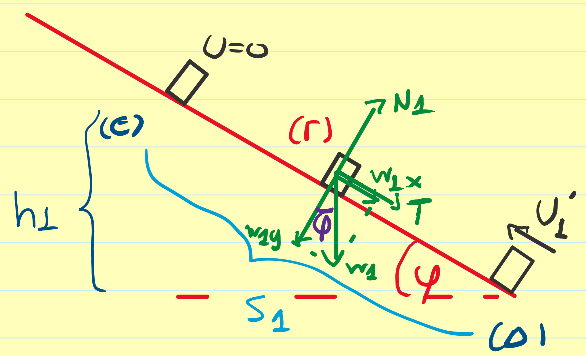
$$\pi_1 = \frac{\Delta K_1}{K_1} \cdot 100\% \Rightarrow \boxed{\pi_1 = -\frac{80}{81} \cdot 100\%}$$

$\boxed{\Delta 5}$



Όταν το σώμα m_2 σταματάει στο κεκλιμένο, δέχεται τις δυνάμεις που φαίνονται στο διπλανό σχήμα:

$\Sigma F_x = w_x - T = 0$ → Παραμένει ακίνητο



$w_{1x} = m_1 \cdot g \cdot ημφ = 10N$

$w_{1y} = m_1 \cdot g \cdot συνφ = 10\sqrt{3}\,N$

$\Sigma F_y = 0 \Rightarrow N_1 = w_{1y} = 10\sqrt{3}\,N$

$T = \mu \cdot N_1 = \frac{\sqrt{3}}{3} \cdot 10\sqrt{3} = 10N$

Το σώμα m_1 διανύει στο κεκλιμένο επίπεδο μέχρι να σταματήσει:

Θ.Μ.Κ.Ε. (Δ→Ε)

$0 - \frac{1}{2} m_1 \cdot υ_1'^2 = -W_{1x} \cdot S_1 - T \cdot S_1 \Rightarrow -\frac{1}{2} 2 \cdot 1^2 = -10 S_1 - 10 S_1$

$\Rightarrow -1 = -20 S_1 \Rightarrow S_1 = 0{,}05\,m$

Και στη συνέχεια παραμένει ακίνητο και αυτό.

$\boxed{\text{Άρα } S_1 < S_2 \text{, δεν συγκρούονται}}$

$ημφ = \frac{h_2}{S_2} \Rightarrow h_2 = S_2 \cdot ημφ \Rightarrow h_2 = \frac{0{,}45}{2}\,m \rightarrow U_2 = m_2 g h_2 = 6 \cdot 10 \cdot \frac{0{,}45}{2} = 13{,}5J$

$ημφ = \frac{h_1}{S_1} \Rightarrow h_1 = S_1 \cdot ημφ \Rightarrow h_1 = \frac{0{,}05}{2}\,m \rightarrow U_1 = m_1 g h_1 = 2 \cdot 10 \cdot \frac{0{,}05}{2} = 0{,}5J$

$E = \cancel{K_{ολ}}^{0} + U_{ολ} = U_1 + U_2 \Rightarrow \boxed{E = 14J}$