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Διαγώνισμα Άλγεβρας Α' Λυκείου
(Λύσεις)

Θέμα Α

(A₁) Σχολικό Βιβλίο, σελίδα 63

- (A₂) α) Λ
β) Σ
γ) Λ
δ) Σ
ε) Λ

- (A₃) α) iii
β) γ

Θέμα Β

(B₁) i) Αφού $-2 < x < 1$ τότε
 $-4 < 2x < 2$

$$\Rightarrow 1 < 2x+5 < 7$$

Αρα $2x+5 > 0$, έγκι $|2x+5| = 2x+5$

Αφού $-2 < x < 1$ τότε

$$-6 < 3x < 3$$

$$-10 < 3x-4 < -1$$

Αρα $3x-4 < 0$, έγκι $|3x-4| = 4-3x$

$$A = 2x+5 - (4-3x) =$$

$$= 2x+5-4+3x =$$

$$= 5x+1$$

ii) Αφού $x^2 \geq 0$ για κάθε $x \in \mathbb{R}$ τότε $-x^2 \leq 0$ \hookrightarrow Ε761
 $-x^2 - 2022 < 0$ για κάθε $x \in \mathbb{R}$

Αφού $\sqrt{x} \geq 0$ για κάθε $x \geq 0$ τότε $\sqrt{x} + 2022 > 0$

$$\text{Άρα } |-x^2 - 2022| = x^2 + 2022$$
$$\hookrightarrow |\sqrt{x} + 2022| = \sqrt{x} + 2022$$

$$B = x^2 + 2022 - (\sqrt{x} + 2022) =$$
$$= x^2 + 2022 - \sqrt{x} - 2022 =$$
$$= x^2 - \sqrt{x}$$

iii) • Αν $x > 0$ \hookrightarrow $y > 0$ τότε $|x| = x$ \hookrightarrow $|y| = y$

$$\Gamma = \frac{x}{x} - \frac{y}{y} =$$

$$= 1 - 1 =$$

$$= 0$$

• Αν $x < 0$ \hookrightarrow $y < 0$ τότε $|x| = -x$ \hookrightarrow $|y| = -y$

$$\Gamma = \frac{-x}{x} - \frac{-y}{y} =$$

$$= -1 - (-1) =$$

$$= 0$$

• Αν $x > 0$ \hookrightarrow $y < 0$ τότε $|x| = x$ \hookrightarrow $|y| = -y$

$$\Gamma = \frac{x}{x} - \frac{-y}{y} =$$

$$= 1 - (-1) =$$

$$= 2$$

• Αν $x < 0$ \hookrightarrow $y > 0$ τότε $|x| = -x$ \hookrightarrow $|y| = y$

$$\Gamma = \frac{-x}{x} - \frac{y}{y} =$$

$$= -1 - 1 =$$

$$= -2$$

$\textcircled{B_2}$ α) $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$
 $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$
 $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$
 $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$
 $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

Άρα έχουμε: $(2\sqrt{2} - 3\sqrt{2}) \cdot (5\sqrt{2} + 6\sqrt{2} - 4\sqrt{2}) =$
 $= (-\sqrt{2}) \cdot (7\sqrt{2}) =$
 $= -7 \cdot \sqrt{2}^2 =$
 $= -7 \cdot 2 =$
 $= -14$

β) $\sqrt{7} \cdot \sqrt{3-\sqrt{2}} \cdot \sqrt{3+\sqrt{2}} =$
 $= \sqrt{7 \cdot (3-\sqrt{2}) \cdot (3+\sqrt{2})} =$
 $= \sqrt{7 \cdot (3^2 - \sqrt{2}^2)} =$
 $= \sqrt{7 \cdot (9-2)} =$
 $= \sqrt{7^2} =$
 $= 7$

Θέμα Γ

$\textcircled{\Gamma_1}$ $a^2 = (1+\sqrt{2})^2 = 1^2 + 2\sqrt{2} + \sqrt{2}^2 = 3 + 2\sqrt{2}$
 $b^2 = (1-\sqrt{2})^2 = 1^2 - 2\sqrt{2} + \sqrt{2}^2 = 3 - 2\sqrt{2}$

$A = a^2 - b^2 = 3 + 2\sqrt{2} - (3 - 2\sqrt{2}) = 3 + 2\sqrt{2} - 3 + 2\sqrt{2} = 4\sqrt{2}$

$\textcircled{\Gamma_2}$ Από $1+\sqrt{2} > 0$ τότε $a > 0$
 Ενώ $1-\sqrt{2} > 0 \Leftrightarrow 1 > \sqrt{2} \Leftrightarrow 1^2 > \sqrt{2}^2 \Leftrightarrow 1 > 2$ Άρα
 Άρα $1-\sqrt{2} < 0$ οπότε $b < 0$
 $B = \sqrt{a^2} - \sqrt{b^2} = |a| - |b| = a - (-b) = a + b = 1 + \sqrt{2} + 1 - \sqrt{2} = 2$

$$\textcircled{\Gamma_3} \quad \sqrt{a^2 - b^2} > \sqrt{a^2} - \sqrt{b^2} \Leftrightarrow$$

$$\Leftrightarrow \sqrt{A} > B \Leftrightarrow$$

$$\Leftrightarrow \sqrt{4\sqrt{2}} > 2 \Leftrightarrow$$

$$\Leftrightarrow (\sqrt{4\sqrt{2}})^2 > 2^2 \Leftrightarrow$$

$$\Leftrightarrow 4\sqrt{2} > 4 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{2} > 1 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{2}^2 > 1^2 \Leftrightarrow$$

$$\Leftrightarrow 2 > 1 \quad \underline{\text{16xύει}}$$

Θέμα Δ

$$\begin{aligned} \textcircled{\Delta_1} \quad x \cdot \varphi \cdot z &= \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2-\sqrt{2+\sqrt{3}}} \cdot \sqrt{2+\sqrt{3}} = \\ &= \sqrt{(2+\sqrt{2+\sqrt{3}}) \cdot (2-\sqrt{2+\sqrt{3}}) \cdot (2+\sqrt{3})} = \\ &= \sqrt{[2^2 - (\sqrt{2+\sqrt{3}})^2] \cdot (2+\sqrt{3})} = \\ &= \sqrt{[4 - (2+\sqrt{3})] \cdot (2+\sqrt{3})} = \\ &= \sqrt{(2-\sqrt{3})(2+\sqrt{3})} = \\ &= \sqrt{2^2 - \sqrt{3}^2} = \\ &= \sqrt{4-3} = \\ &= 1 \end{aligned}$$

$$\textcircled{\Delta_2} \quad \text{i) } |2|x|-1| = 3$$

$$\Rightarrow 2|x|-1 = 3$$

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$$2|x|-1 = -3$$

$$\Rightarrow 2|x| = 4$$

$$2|x| = -2$$

$$\Rightarrow |x| = 2$$

$$|x| = -1$$

$$\Rightarrow x = 2 \quad \text{ή} \quad x = -2$$

Αδύναμ

$$ii) |x-1| \cdot |x-2| = |x-1|$$

$$\Rightarrow |(x-1)(x-2)| = |x-1|$$

$$\Rightarrow (x-1)(x-2) = x-1 \quad \text{ή}$$

$$\Rightarrow (x-1)(x-2) - (x-1) = 0$$

$$\Rightarrow (x-1) \cdot (x-2-1) = 0$$

$$\Rightarrow (x-1) \cdot (x-3) = 0$$

$$\Rightarrow x-1=0 \quad \text{ή} \quad x-3=0$$

$$\Rightarrow x=1 \quad \text{ή} \quad x=3$$

$$(x-1)(x-2) = -(x-1)$$

$$(x-1)(x-2) + (x-1) = 0$$

$$(x-1)(x-2+1) = 0$$

$$(x-1)(x-1) = 0$$

$$x-1 = 0$$

$$x=1$$

$$iii) \sqrt{x^2 - 10x + 25} = 3x - 5$$

$$\text{Πρέπει } x^2 - 10x + 25 \geq 0 \Leftrightarrow (x-5)^2 \geq 0 \quad \text{16xύει}$$

$$\hookrightarrow 3x - 5 \geq 0 \Leftrightarrow 3x \geq 5 \Leftrightarrow x \geq \frac{5}{3}$$

$$\text{Οπότε } \sqrt{(x-5)^2} = 3x - 5$$

$$\Rightarrow |x-5| = 3x - 5$$

$$\Rightarrow x-5 = 3x-5 \quad \text{ή}$$

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$$x-5 = -3x+5$$

$$\Rightarrow 2x = 0$$

$$4x = 10$$

$$\Rightarrow x = 0$$

$$x = \frac{5}{2}$$

Απορρίπτεται

$$\textcircled{\Delta_3} |k| - 3\lambda + 1 + |\mu^2 - 6|\mu| + 9| = 0$$

$$\text{Πρέπει } |k| - 3\lambda + 1 = 0$$

$$\Leftrightarrow |k| = 3\lambda - 1$$

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$$\mu^2 - 6|\mu| + 9 = 0$$

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$$|\mu|^2 - 6|\mu| + 9 = 0$$

$$(|\mu| - 3)^2 = 0$$

$$i) \text{ Αφού } |k| \geq 0 \text{ τότε πρέπει } 3\lambda - 1 \geq 0$$

$$\Rightarrow 3\lambda \geq 1$$

$$\Rightarrow \lambda \geq \frac{1}{3}$$

$$ii) (|\mu|-3)^2=0 \Rightarrow$$

$$\Rightarrow |\mu|-3=0$$

$$\Rightarrow |\mu|=3$$

$$\Rightarrow \mu=3 \text{ ή } \mu=-3$$

Όμως $|\mu-3|>0$ δηλαδή $\mu-3 \neq 0 \Rightarrow \mu \neq 3$

Επομένως $\mu=-3$