

16/10/2022

# Διαγώνισμα Άλγεβρα Α' Λυκείου

## Λύσεις

### Θέμα Α

Ⓐ) Σχολικό Βιβλίο, σελίδα 54

Ⓐ<sub>2</sub>

- 1) ii
- 2) iv
- 3) iii
- 4) iv

Ⓐ<sub>3</sub>

- i) Σ
- ii) Λ
- iii) Λ
- iv) Λ
- v) Λ

### Θέμα Β

Ⓑ<sub>1</sub>) i)  $(a+b)^3 - (a-b)^3 - 6a^2b = a^3 + 3a^2b + 3ab^2 + b^3 - a^3 + 3a^2b - 3ab^2 + b^3 - 6a^2b =$   
 $= 2b^3$

ii)  $(a-1)^2 - 2(a-1)(a+1) + (a+1)^2 = a^2 - 2a + 1 - 2(a^2 - 1) + a^2 + 2a + 1 =$   
 $= a^2 + 1 - 2a^2 + 2 + a^2 + 1 =$   
 $= 4$

$$\textcircled{B_2} \quad 1) \quad i) \quad a^2 - b^2 = 2a + b - 2b - a =$$

$$= a - b$$

$$ii) \quad \text{Αφού} \quad a^2 - b = a - b$$

$$\Rightarrow (a-b)(a+b) - (a-b) = 0$$

$$\Rightarrow (a-b)(a+b-1) = 0$$

$$\Rightarrow a-b=0 \quad \text{ή} \quad a+b-1=0$$

$$\Rightarrow a=b \quad \Rightarrow a+b=1$$

Απάντη

$$2) \quad A = a^2 + b^2 = 2a + b + 2b + a =$$

$$= 3a + 3b =$$

$$= 3(a+b) =$$

$$= 3 \cdot 1 =$$

$$= 3$$

Οερα Γ

$$\textcircled{\Gamma_1} \quad i) \quad \frac{x^3 - x^2 + x}{x^3 + 1} = \frac{x(x^2 - x + 1)}{(x+1)(x^2 - x + 1)} = \frac{x}{x+1}$$

$$ii) \quad \frac{(x+2)(2x-1)(x-1) + x - 4}{x^2 - 2} = \frac{(2x^2 - x + 4x - 2)(x-1) + x - 4}{x^2 - 2} =$$

$$= \frac{2x^3 - 2x^2 + 3x^2 - 3x - 2x + 2 + x - 4}{x^2 - 2} = \frac{2x^3 + x^2 - 4x - 2}{x^2 - 2} =$$

$$= \frac{x^2(2x+1) - 2(2x+1)}{x^2 - 2} = \frac{(2x+1)(x^2 - 2)}{x^2 - 2} = 2x+1$$



$$\begin{aligned} \text{iii)} \quad \left(a - \frac{1}{a}\right)^2 \cdot \frac{a^3 + a^2}{(a+1)^3} &= \left(\frac{a^2 - 1}{a}\right)^2 \cdot \frac{a^2 \cdot (a+1)}{(a+1)^3} = \\ &= \left[\frac{(a-1)(a+1)}{a}\right]^2 \cdot \frac{a^2}{(a+1)^2} = \frac{(a-1)^2 (a+1)^2}{a^2} \cdot \frac{a^2}{(a+1)^2} = \\ &= (a-1)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{\Gamma_2} \quad A &= \frac{x(x^4 y^5)^4}{x^{-9} \cdot y^{-1} (x^{-2} y^{-3})^5} = \frac{x \cdot x^{16} \cdot y^{20}}{x^{-9} \cdot y^{-1} \cdot x^{-10} \cdot y^{-15}} = \frac{x^{17} \cdot y^{20}}{x^{-19} \cdot y^{-16}} = \\ &= x^{36} \cdot y^{36} = (x \cdot y)^{36} \stackrel{*}{=} 1^{36} = 1 \end{aligned}$$

\* Αφού  $x, y$  αντίστροφοι τότε  $x \cdot y = 1$

$$\textcircled{\Gamma_3} \quad -1 \leq \frac{2x}{x^2+1} \quad \Leftrightarrow \quad \frac{2x}{x^2+1} \leq 1$$

Αφού  $x^2+1 > 0$  για κάθε  $x \in \mathbb{R}$ , τότε:

$$-(x^2+1) \leq 2x \quad \Leftrightarrow \quad 2x \leq x^2+1$$

$$\begin{aligned} \Leftrightarrow x^2+1+2x &\geq 0 & \Leftrightarrow x^2-2x+1 &\geq 0 \\ \Leftrightarrow (x+1)^2 &\geq 0 & \Leftrightarrow (x-1)^2 &\geq 0 \\ \text{16xύει} & & \text{16xύει} & \end{aligned}$$

### Θέμα Δ

$$\textcircled{\Delta_1} \quad \text{i)} \quad \text{Αφού } 2 \leq a \leq 4 \quad \textcircled{1} \quad \Leftrightarrow \quad 1 \leq b \leq 6$$

$$\Leftrightarrow 2 \leq 2b \leq 12 \quad \textcircled{2}$$

τότε από  $\textcircled{1} + \textcircled{2} \Rightarrow 4 \leq a+2b \leq 16$

$$\begin{aligned} \text{ii)} \quad \text{Αφού } 2 \leq a \leq 4 \quad \textcircled{1} \quad \Leftrightarrow \quad 1 \leq b \leq 6 &\Leftrightarrow -1 \geq -b \geq -6 \\ &\Leftrightarrow -6 \leq -b \leq -1 \quad \textcircled{2} \end{aligned}$$



$$\text{Απο } \textcircled{1} + \textcircled{2} \Rightarrow -4 \leq a - b \leq 3$$

$$\text{iii) Αφού } 2 \leq a \leq 4 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2} \geq \frac{1}{a} \geq \frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{4} \leq \frac{1}{a} \leq \frac{1}{2} \quad \textcircled{1}$$

$$\text{κ' } 1 \leq b \leq 6 \Leftrightarrow$$

$$\Leftrightarrow 1 \geq \frac{1}{b} \geq \frac{1}{6} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{6} \leq \frac{1}{b} \leq 1 \quad \textcircled{2}$$

$$\text{τότε απο } \textcircled{1} + \textcircled{2} \Rightarrow \frac{1}{4} + \frac{1}{6} \leq \frac{1}{a} + \frac{1}{b} \leq \frac{1}{2} + 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{5}{12} \leq \frac{1}{a} + \frac{1}{b} \leq \frac{3}{2}$$

$$\text{iv) Αφού } 2 \leq a \leq 4 \quad \textcircled{1}$$

$$\text{κ' } 1 \leq b \leq 6 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{6} \leq \frac{1}{b} \leq 1 \quad \textcircled{2}$$

$$\text{τότε απο } \textcircled{1} \cdot \textcircled{2} \Rightarrow \frac{2}{6} \leq \frac{a}{b} \leq 4 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{3} \leq \frac{a}{b} \leq 4$$

$$\text{v) Αφού } 2 \leq a \leq 4 \Leftrightarrow$$

$$\Leftrightarrow 4 \leq a^2 \leq 16 \quad \textcircled{1}$$

$$\text{κ'}$$

$$-4 \leq b \leq 6 \Leftrightarrow$$

$$\Leftrightarrow 1 \leq b^2 \leq 36 \quad \textcircled{2}$$

$$\text{τότε απο } \textcircled{1} + \textcircled{2} \rightarrow 5 \leq a^2 + b^2 \leq 52$$

$$\begin{aligned}
 \textcircled{\Delta_2} \quad A &= x^2 - 10xy + 27y^2 - 8y + 8 = \\
 &= x^2 - 10xy + 25y^2 + 2y^2 - 8y + 8 = \\
 &= (x - 5y)^2 + 2 \cdot (y^2 - 4y + 4) = \\
 &= (x - 5y)^2 + 2(y - 2)^2 = \\
 &= (x - 5y)^2 + [\sqrt{2}(y - 2)]^2
 \end{aligned}$$

$$A = 0 \Leftrightarrow (x - 5y)^2 + [\sqrt{2}(y - 2)]^2 = 0$$

$$\Leftrightarrow x - 5y = 0 \quad \text{και} \quad \sqrt{2}(y - 2) = 0$$

$$\Leftrightarrow x = 5y$$

$$\Leftrightarrow y - 2 = 0$$

$$\Leftrightarrow y = 2$$

$$\text{Αρα} \quad x = 5 \cdot 2 \Leftrightarrow x = 10$$

$$\textcircled{\Delta_3} \quad \text{'Εστω} \quad \text{ου} \quad b = 2 \quad \text{Τότε} \quad \frac{1}{a} + \frac{1}{2} = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{a} = 0 \quad \text{'Ατοπο!!}$$

$$\text{Αρα} \quad b \neq 2$$