

Α' ΜΕΡΟΣ: ΜΑΘΗΜΑΤΙΚΑ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ

ΘΕΜΑ Α

- (A1) α) Ψωστό
β) Ψωστό
γ) Λάθος
δ) Λάθος
ε) Λάθος

$$(A2) \text{ i) } \lim_{x \rightarrow 1} \frac{3-x+\frac{2}{x-2}}{4-\frac{2x+2}{x^2}} = \lim_{x \rightarrow 1} \frac{\frac{3x-6-x^2+2x+2}{x-2}}{\frac{4x^2-2x-2}{x^2}} = \lim_{x \rightarrow 1} \frac{x \cdot (-x^2+5x-4)}{(x-2)(4x^2-2x-2)} =$$

$$= \lim_{x \rightarrow 1} \frac{-x \cdot (x-1)(x-4)}{2(x-2)(2x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{4-x}{2(x-2)(2x+1)} = \frac{3}{2 \cdot (-1) \cdot 3} = \underline{\underline{\frac{1}{2}}}$$

$$\text{ii) } \lim_{x \rightarrow 1} \frac{\sqrt{8x+1} + \sqrt{x^2+3} - 5}{\sqrt{3x-2} - 1} = \lim_{x \rightarrow 1} \left(\frac{\sqrt{8x+1} - 3}{\sqrt{3x-2} - 1} + \frac{\sqrt{x^2+3} - 2}{\sqrt{3x-2} - 1} \right) =$$

$$= \lim_{x \rightarrow 1} \left[\frac{(8x+1-9)(\sqrt{3x-2}+1)}{(3x-2-1)(\sqrt{8x+1}+3)} + \frac{(x^2+3-4)(\sqrt{3x-2}+1)}{(3x-2-1) \cdot (\sqrt{x^2+3}+2)} \right] =$$

$$= \lim_{x \rightarrow 1} \left[\frac{8 \cdot (x-1) \cdot (\sqrt{3x-2}+1)}{3 \cdot (x-1) \cdot (\sqrt{8x+1}+3)} + \frac{(x-1)(x+1) \cdot (\sqrt{3x-2}+1)}{3(x-1)(\sqrt{x^2+3}+2)} \right] =$$

$$= \lim_{x \rightarrow 1} \left[\frac{8 \cdot (\sqrt{3x-2}+1)}{3 \cdot (\sqrt{8x+1}+3)} + \frac{(x+1) \cdot (\sqrt{3x-2}+1)}{3(\sqrt{x^2+3}+2)} \right] = \frac{8 \cdot 2}{3 \cdot 6} + \frac{2 \cdot 2}{3 \cdot 4} = \dots = \underline{\underline{\frac{11}{9}}}$$

iii) $\bullet \lim_{x \rightarrow -1} (x^2 + 5x + 5) = 1 - 5 + 5 = 1 > 0$, άρα $x^2 + 5x + 5 > 0$ για $x < -1$

$\bullet \lim_{x \rightarrow -1} (x^3 + 5x + 4) = -1 - 5 + 4 = -2 < 0$, άρα $x^3 + 5x + 4 > 0$ για $x < -1$

Άρα: $\lim_{x \rightarrow -1} \frac{|x^2 + 5x + 5| + x}{|x^3 + 5x + 4| - 2} = \lim_{x \rightarrow -1} \frac{x^2 + 5x + 5 + x}{-x^3 - 5x - 4 - 2} =$

$= \lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{-x^3 - 5x - 6} = \lim_{x \rightarrow -1} \frac{(x+1)(x+5)}{(x+1)(-x^2+x-6)} =$

x	-1	0	-5	-6	-1
		$+1$	-1	6	
	-1	$+1$	-6	6	

$= \lim_{x \rightarrow -1} \frac{x+5}{-x^2+x-6} = \frac{4}{-8} = -\frac{1}{2}$

ΘΕΜΑ Β

(A) $A_f = (1, 3] \cup (3, 5) \cup (5, 7] \cup \{7\} \cup (7, 9] = (1, 5) \cup (5, 9]$

(B) i) $\lim_{x \rightarrow 1} f(x) = -2$ ii) $\lim_{x \rightarrow 3^-} f(x) = 1$
 $\lim_{x \rightarrow 3^+} f(x) = 2$ } $\Rightarrow \nexists \lim_{x \rightarrow 3} f(x)$

iii) $\lim_{x \rightarrow 7^-} f(x) = 2$ } $\Rightarrow \nexists \lim_{x \rightarrow 7} f(x)$
 $\lim_{x \rightarrow 7^+} f(x) = 4$ }
 iv) $\lim_{x \rightarrow 9} f(x) = 3$

v) $\lim_{x \rightarrow 6} \sqrt{f(x) + 9} = \sqrt{0^2 + 9} = \sqrt{9} = 3$

vi) $\lim_{x \rightarrow 5} \frac{1}{f(x)} = \frac{1}{3}$

$$\text{vii)} \lim_{x \rightarrow 2} \frac{|f(x) \cdot (x-4)|}{x f(x)} \stackrel{0}{=} \lim_{x \rightarrow 2} \frac{|f(x)|}{f(x)} \frac{|x-4|}{x} =$$

$$= \begin{cases} \lim_{x \rightarrow 2^-} \frac{|f(x)|}{f(x)} \frac{|x-4|}{x} = \lim_{x \rightarrow 2^-} \frac{-f(x)}{f(x)} \frac{(4-x)}{x} = \lim_{x \rightarrow 2^-} \frac{x-4}{x} = -1 \\ \lim_{x \rightarrow 2^+} \frac{|f(x)|}{f(x)} \frac{|x-4|}{x} = \lim_{x \rightarrow 2^+} \frac{f(x)}{f(x)} \frac{4-x}{x} = \lim_{x \rightarrow 2^+} \frac{4-x}{x} = 1 \end{cases}$$

αρα δεν υπάρχει το όριο

$$\text{viii)} \lim_{x \rightarrow 4} \frac{\sqrt{f(x)+9} - 2f(x)}{f(x)-4} \stackrel{0}{=} \lim_{x \rightarrow 4} \frac{f^2(x)+9-4f^2(x)}{(f(x)-4)(\sqrt{f(x)+9}+2f(x))} =$$

$$= \lim_{x \rightarrow 4} \frac{-3 \cdot (f(x)-4)(f(x)+4)}{(f(x)-4)(\sqrt{f(x)+9}+2f(x))} =$$

$$= \lim_{x \rightarrow 4} \frac{-3 \cdot (f(x)+4)}{\sqrt{f(x)+9}+2f(x)} = \frac{-3 \cdot (4+4)}{\sqrt{4^2+9}+2 \cdot 4} = \frac{-24}{8+8} = \underline{\underline{-\frac{3}{2}}}$$

$$\text{(B2) a)} \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{\sqrt[3]{x}-1}{x^2-x} = \lim_{x \rightarrow 1^-} \frac{x-1}{x(x-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} =$$

$$= \lim_{x \rightarrow 1^-} \frac{1}{x(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \underline{\underline{\frac{1}{3}}}$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{-x^3+4x^2-3x}{(1-\sqrt{x+3})(3x^2-12x+9)} = \frac{-x(x^2-4x+3)}{(1-\sqrt{x+3}) \cdot 3(x^2-4x+3)} =$$

$$= \lim_{x \rightarrow 1^+} \frac{-x}{3(1-\sqrt{x+3})} = \underline{\underline{\frac{1}{3}}}$$

Αρα: $\lim_{x \rightarrow 1} f(x) = \frac{1}{3}$

$$e) \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{-x^3 + 4x^2 - 3x}{(1 - \sqrt{x+3}) \cdot (3x^2 - 12x + 9)} \stackrel{(a)}{=} \underline{\underline{\quad}}$$

$$= \lim_{x \rightarrow 3^-} \frac{-x}{3 \cdot (1 - \sqrt{x+3})} = \frac{-3}{3 \cdot (1 - \sqrt{6})}$$

$$= \frac{1}{\sqrt{6} - 1} = \underline{\underline{\frac{\sqrt{6} + 1}{5}}}$$

$$\bullet \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{\sqrt{x^2+2} - \sqrt{11}}{x^2 - 5x + 6} = \lim_{x \rightarrow 3^+} \frac{x^2 + 2 - 11}{(x-2)(x-3) \cdot (\sqrt{x^2+2} + \sqrt{11})} =$$

$$= \lim_{x \rightarrow 3^+} \frac{(x-3)(x+3)}{(x-2)(x-3) \cdot (\sqrt{x^2+2} + \sqrt{11})} = \lim_{x \rightarrow 3^+} \frac{x+3}{(x-2) \cdot (\sqrt{x^2+2} + \sqrt{11})} =$$

$$= \frac{6}{1 \cdot (\sqrt{11} + \sqrt{11})} = \frac{6}{2\sqrt{11}} = \underline{\underline{\frac{3\sqrt{11}}{11}}}$$

Άρα $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$, οπότε δεν υπάρχει το $\lim_{x \rightarrow 3} f(x)$

ΘΕΜΑ Γ

Γ1 Θεωρία, σχολικό \rightarrow σελ. 134

Γ4 (δ)

Γ2 Θεωρία, σχολικό \rightarrow σελ. 135

Γ5 1. Λάθος

Γ3 α) ψ

2. Ξωστό

β) Έστω: $P(x) = x^2 + x + 1 : 2^{\circ}$ βαθμού

3. Ξωστό

$Q(x) = -x^2 + x + 1 : 2^{\circ}$..

4. Λάθος

άρα $P(x) + Q(x) = 2x + 2 : 1^{\circ}$..

5. Λάθος

ΘΕΜΑ Δ

$\begin{array}{r} 3x^4 - 4x^3 + 5x^2 - 8x + 9 \\ -3x^4 + 6x^3 - 3x^2 \\ \hline 2x^3 + 2x^2 - 8x + 9 \\ -2x^3 + 4x^2 - 2x \\ \hline 6x^2 - 10x + 9 \\ -6x^2 + 12x - 6 \\ \hline 2x + 3 \end{array}$	$\begin{array}{r} x^2 - 2x + 1 \\ 3x^2 + 2x + 6 \end{array}$
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β) $3x^4 - 4x^3 + 5x^2 - 8x + 9 = (x-1)^2 \cdot (3x^2 + 2x + 6) + 2x + 3$

γ) Το $(x-1)^2$ δεν είναι διαιρέτης του $3x^4 - 4x^3 + 5x^2 - 8x + 9$
 διότι $v(x) = 2x + 3 \neq 0$

Δ2) Είναι: $P(x) = \lambda(\lambda^2 - 5\lambda + 6)x^4 - \lambda(9 - \lambda^2)x^3 + \lambda(\lambda^3 - 27)x^2 + (\lambda^2 - 3\lambda)x + \lambda$
 $P(x) = \lambda(\lambda - 2)(\lambda - 3)x^4 - \lambda(9 - \lambda^2)x^3 + \lambda(\lambda^3 - 27)x^2 + (\lambda^2 - 3\lambda)x + \lambda$

Έστω: $\lambda(\lambda - 2)(\lambda - 3) = 0 \Leftrightarrow \lambda = 0 \vee \lambda = 2 \vee \lambda = 3$

- Αν $\lambda \neq 0$ και $\lambda \neq 2$ και $\lambda \neq 3$, τότε το $P(x)$ είναι 4^ο βαθμού
- Αν $\lambda = 0$, τότε: $P(x) = 0$, οπότε δεν ορίζεται βαθμός
- Αν $\lambda = 2$, τότε: $P(x) = -10x^3 - 38x^2 - 2x + 2$: 3^ο βαθμού
- Αν $\lambda = 3$, τότε: $P(x) = 3$: μηδενικού βαθμού

Δ3) Έχουμε: $P(x) = x^{2021} + a \cdot x^{2020} + b \cdot x^{1821} + 5$

Δίνεται: $P(1) = 0 \Rightarrow 1 + a + b + 5 = 0 \Rightarrow a + b = -6$
 $P(-1) = 2 \Rightarrow -1 + a - b + 5 = 2 \Rightarrow a - b = -2$

$a = -4$
 $b = -2$