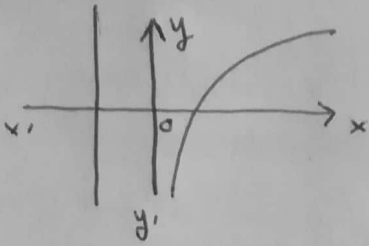


ΛΥΣΕΙΣ Α ΛΥΚΕΙΟΥ 9/5/2021

ΖΗΤΗΜΑ Α

A2 1. Λ 2. Λ 3. Λ 4. Σ

A3 Ψ



ΤΟ ΠΟΛΥ ΕΝΑ ΣΗΜΕΙΟ

ΖΗΤΗΜΑ Β

B1 i) $A_f = \mathbb{R}$

ii) ηρμεν $x^2 - x - 2 \neq 0$

$$\Delta = 1 + 8 = 9 \quad x = \begin{cases} \frac{1+3}{2} = 4/2 = 2 \\ \frac{1-3}{2} = -2/2 = -1 \end{cases} \quad A_f = \mathbb{R} - \{-1, 2\}$$

B2 i) ηρμεν: $\begin{cases} x^2 + 5x \neq 0 \rightarrow x \neq 0 \text{ κ' } x \neq -5 \\ x^2 - 2x + 3 \neq 0 \end{cases}$

$$\Delta = 4 + 12 = 16 \quad x_{1,2} = \begin{cases} \frac{2+4}{2} = 3 \\ \frac{2-4}{2} = -1 \end{cases}$$

∴ $A_f = \mathbb{R} - \{0, -5, -1, 3\}$

ii) ηρμεν: $|2x-1| - 5 \neq 0 \quad A_f = \mathbb{R} - \{3, -2\}$

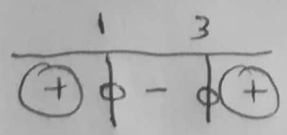
$$|2x-1| = 5 \Leftrightarrow \begin{cases} 2x-1 = 5 & \text{ή} \\ 2x-1 = -5 \end{cases}$$

$$\begin{matrix} 2x = 6 & & 2x = -4 \\ x = 3 & & x = -2 \end{matrix}$$

B3 ηρμεν $x^2 - 4x + 3 \geq 0$ και $-x^2 + 4x + 12 \geq 0$

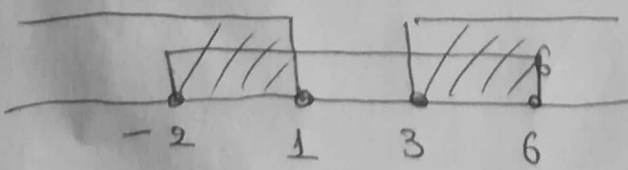
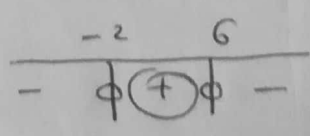
• $\Delta = 16 - 12 = 4$

$$x = \begin{cases} \frac{4-2}{2} = 1 \\ \frac{4+2}{2} = 3 \end{cases}$$



$\Delta = 16 + 48 = 64$

$$x_{1,2} = \frac{-4 \pm 8}{-2} = \begin{cases} \frac{-4-8}{-2} = \frac{-12}{-2} = 6 \\ \frac{-4+8}{-2} = \frac{4}{-2} = -2 \end{cases}$$



∴ $A_f = [-2, 1] \cup [3, 6]$

(1)

ЗНАЧЕНИЯ Γ

Γ₁ | f(0) = 6 → (0, 6)

① f(x) = 0 ⇔ x² - 5x + 6 = 0 Δ = 25 - 24 = 1 x = $\begin{cases} \frac{5+1}{2} = 3 \\ \frac{5-1}{2} = 2 \end{cases}$
 (3, 0) и (2, 0)

② x² - 5x + 6 > -x² - 2x + 15 ⇔ 2x² - 3x - 9 > 0

Δ = 9 + 72 = 81 x = $\frac{3 \pm 9}{4} = \begin{cases} \frac{12}{4} = 3 \\ \frac{-6}{4} = -\frac{3}{2} \end{cases}$
 $\begin{array}{c} -3/2 \quad 3 \\ \hline + \quad | \quad - \quad | \quad + \end{array}$ ∴ x ∈ (-∞, -3/2) ∪ (3, +∞)

Γ₂ | ① 2x² - 5x + 3 = 2(x - 3/2)(x - 1) = (2x - 3)(x - 1)

Δ = 25 - 24 = 1

x = $\begin{cases} \frac{5+1}{4} = 6/4 = 3/2 \\ \frac{5-1}{4} = 1 \end{cases}$

• x² - 4x + 3 = (x - 3)(x - 1)

Δ = 16 - 12 = 4

x = $\frac{4 \pm 2}{2} = \begin{cases} 6/2 = 3 \\ 1 \end{cases}$

② η притоки: x² - 4x + 3 ≠ 0 ∴ A f = ℝ - {1, 3}

③ h(x) = $\frac{(2x-3)(\cancel{x-3})(\cancel{x-1})}{(\cancel{x-3})(\cancel{x-1})} = 2x-3$

④ |2x - 3| = 5 ⇔ $\begin{array}{l} 2x - 3 = 5 \quad \eta \quad 2x - 3 = -5 \\ 2x = 8 \quad \quad \quad 2x = -2 \\ x = 4 \quad \quad \quad x = -1 \end{array}$

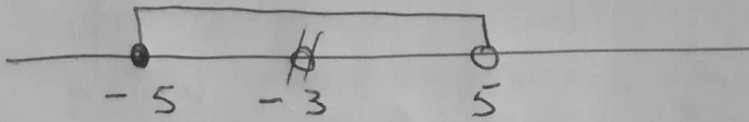
Δ1

①

$$25 - x^2 \geq 0 \Leftrightarrow x^2 \leq 25 \Leftrightarrow |x| \leq 5 \Leftrightarrow -5 \leq x \leq 5$$

$$|x-1|=4 \Leftrightarrow x-1=4 \quad \vee \quad x-1=-4$$

$$x=5 \quad \vee \quad x=-3$$

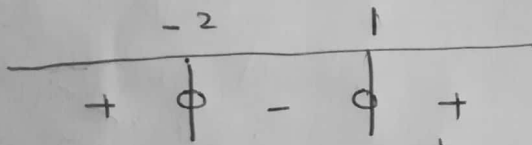


αα $A_f = [-5, -3) \cup (-3, 5]$

② $f(3) = \frac{\sqrt{25-9}}{|3-1|-4} = \frac{\sqrt{16}}{2-4} = \frac{4}{-2} = -2$

$$x^4 + x^2 - 2 > 0 \Leftrightarrow y^2 + y - 2 > 0 \quad \Delta = 1+8=9$$

$$y = \begin{cases} \frac{-1+3}{2} = \frac{2}{2} = 1 \\ \frac{-1-3}{2} = \frac{-4}{2} = -2 \end{cases}$$



$y < -2$ $y > 1$

$x^2 < -2$ $x^2 > 1 \Leftrightarrow \boxed{x > 1} \vee \boxed{x < -1}$

Αδυνα

③

$$\frac{\sqrt{25-x^2}}{|x-1|-4} = \frac{\sqrt{4|x|+30}}{|x-1|-4} \Leftrightarrow \sqrt{25-x^2} = \sqrt{4|x|+30} \Leftrightarrow 25-x^2 = 4|x|+30$$

$$\Leftrightarrow x^2 + 4|x| + 5 = 0 \Leftrightarrow |x|^2 + 4|x| + 5 = 0 \quad \text{ΑΔΥΝΑΤΗ}$$

ααα $|x|^2 + 4|x| + 5 > 0 \quad \forall x \in \mathbb{R}$

Δ2

$$g(x) \underbrace{(g^2(x)+1)}_{>0} = x^2 - 3x - 4 \Leftrightarrow g(x) = \frac{x^2 - 3x - 4}{\underbrace{g^2(x)+1}_{>0}}$$

αα $g(x) < 0$ οταν $x \in (-1, 4)$

(3)