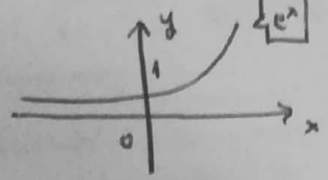
ΛΥΣΗ ΔΙΑΓΩΝΙΣΜΑΤΟΣ Β ΛΥΚΕΙΟΥ

ΜΑΘΗΜΑΤΙΚΑ 5/2/2023

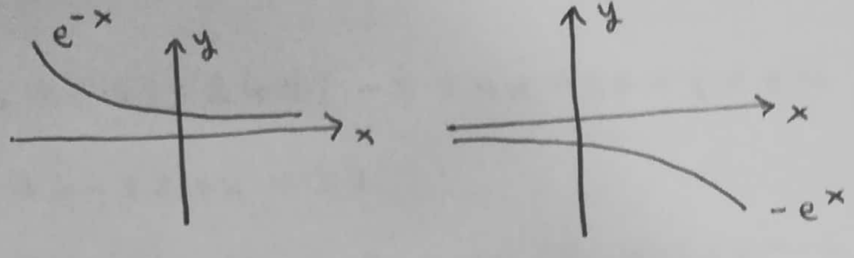
ΘΕΜΑ Α

A1 α)



β) i) $A_f = \mathbb{R}$ ii) $f(A) = (0, +\infty)$ iii) ↑ iv) $(0,1)$ v) $x'x$

γ)



A2 α) Σ β) Λ γ) Σ

ΘΕΜΑ Β

B1 α) $(3^3)^{4x} = (3^2)^{x+1} \Leftrightarrow 3^{12x} = 3^{2x+2} \Leftrightarrow 12x = 2x+2 \Leftrightarrow 10x = 2 \Leftrightarrow \boxed{x = \frac{1}{5}}$

β) πρέπει: $\begin{cases} 2x+7 \geq 0 \Leftrightarrow 2x \geq -7 \Leftrightarrow x \geq -\frac{7}{2} \\ x+2 \geq 0 \Leftrightarrow x \geq -2 \end{cases} \rightarrow \boxed{x \geq -2}$

▸ $\sqrt{2x+7} = x+2 \Leftrightarrow 2x+7 = x^2+4x+4 \Leftrightarrow x^2+2x-3 = 0$

$\Delta = 4+12 = 16$, $x = \begin{cases} \frac{-2+4}{2} = \boxed{1} \text{ δεκτή} \\ \frac{-2-4}{2} = \frac{-6}{2} = -3 \text{ απορ. } x \geq -2 \end{cases}$

γ) $2 \cdot 4^x - 5 \cdot 2^x + 2 = 0 \Leftrightarrow 2y^2 - 5y + 2 = 0$ $\Delta = 25 - 16 = 9$

$y = \begin{cases} \frac{5+3}{4} = 2 \rightarrow 2^x = 2 \Leftrightarrow \boxed{x = 1} \\ \frac{5-3}{4} = \frac{1}{2} \rightarrow 2^x = 2^{-1} \Leftrightarrow \boxed{x = -1} \end{cases}$

B2 α) $3^{2x} - 2 \cdot 3^x - 3 < 0 \Leftrightarrow y^2 - 2y - 3 < 0$, $\Delta = 4+12 = 16$

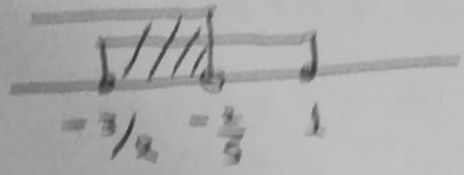
$y = \begin{cases} \frac{2+4}{2} = 3 \\ \frac{2-4}{2} = -1 \end{cases}$

	-1		3	
$+$	ϕ	$-$	ϕ	$+$

$-1 < y < 3 \Leftrightarrow -1 < 3^x < 3 \Leftrightarrow 3^x < 3^1 \Leftrightarrow \boxed{x < 1}$

δ) $\begin{cases} 2x+3 \geq 0 \Leftrightarrow x \geq -3/2 \\ 1-3x \geq 0 \Leftrightarrow -3x \geq -1 \Leftrightarrow x \leq 1 \end{cases} \Leftrightarrow -3/2 \leq x \leq 1$

▸ $\sqrt{2x+3} < \sqrt{1-3x} \Leftrightarrow 2x+3 < 1-3x \Leftrightarrow 5x < -2 \Leftrightarrow x < -\frac{2}{5}$



Άρα $x \in \left[-\frac{3}{2}, -\frac{2}{5}\right)$

<u>ΘΕΜΑ Γ</u>

$\boxed{Γ1}$ ▸ $f(-2) = 24 \Leftrightarrow -8 + 4α - 2β + γ = 24$

$\Leftrightarrow 4α - 2β + γ = 32$

▸ $f(1) = 0 \Leftrightarrow 1 + α + β + γ = 0 \Leftrightarrow α + β + γ = -1$

▸ $ημπ = 0$, $συν\frac{2π}{3} = συν\left(π - \frac{π}{3}\right) = -συν\frac{π}{3} = -\frac{1}{2}$

$f(0) = -16\left(-\frac{1}{2}\right) \Leftrightarrow \boxed{γ = 8}$

$\begin{cases} 4α - 2β + 8 = 32 \\ α + β + 8 = -1 \end{cases} \Leftrightarrow \begin{cases} 4α - 2β = 24 \\ α + β = -9 \end{cases} \Leftrightarrow \begin{cases} 2α - β = 12 \\ α + β = -9 \end{cases} +$

$3α = 3 \Leftrightarrow \boxed{α = 1}$

$\boxed{β = -10}$

$\boxed{Γ2}$ $f(x) = x^3 + x^2 - 10x + 8$

1	1	-10	8	1
↓	1	2	-8	
1	2	-8	0	

$f(x) = (x-1)(x^2+2x-8)$

$x = 1$

$Δ = 4 + 32 = 36$

$x = \frac{-2 \pm 6}{2} \begin{cases} \frac{-2+6}{2} = 2 \\ \frac{-2-6}{2} = -4 \end{cases}$

x		-4		1		2	
$x-1$	$-$		$-$	0	$+$		$+$
x^2+2x-8	$+$	0	$-$		$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

$f(x) > 0$ όταν $x \in (-4, 1) \cup (2, +\infty)$

$\boxed{\Gamma_3}$ $f(\eta\mu x) = (\eta\mu x - 1)(\eta\mu x - 2)(\eta\mu x + 4)$

• Είναι: $-1 \le \eta\mu x \le 1 \Leftrightarrow \boxed{\eta\mu x - 1 \le 0}$

$-1 \le \eta\mu x \le 1 \overset{-2}{\Leftrightarrow} -3 \le \boxed{\eta\mu x - 2 \le -1 < 0}$

$-1 \le \eta\mu x \le 1 \overset{+4}{\Leftrightarrow} 3 \le \eta\mu x + 4 \le 5 \Rightarrow \boxed{\eta\mu x + 4 > 0}$

Τελικά, $f(\eta\mu x) = \underbrace{(\eta\mu x - 1)}_{\le 0}\underbrace{(\eta\mu x - 2)}_{< 0}\underbrace{(\eta\mu x + 4)}_{> 0} \ge 0, \ \forall x \in \mathbb{R}$

$\boxed{\Gamma_4}$ $\dfrac{x+4}{(x+4)(x-2)(x-1)} \ge \dfrac{2}{\cancel{x^3}+x^2-\cancel{10x}+8-\cancel{x^3}+x^2+\cancel{10x}+8-18}$

Περιορισμοί
$x \ne -4$
$x \ne 2$
$x \ne 1$
$x \ne -1$

$\Leftrightarrow \dfrac{x+4}{(x+4)(x-2)(x-1)} \ge \dfrac{2}{2x^2-2} \Leftrightarrow$

$\Leftrightarrow \dfrac{1}{(x-2)(x-1)} \ge \dfrac{1}{(x-1)(x+1)} \Leftrightarrow$

$\Leftrightarrow \dfrac{x+1-x+2}{(x-2)(x-1)(x+1)} \ge 0 \Leftrightarrow \dfrac{3}{(x-2)(x-1)(x+1)} \ge 0 \Leftrightarrow (x-2)(x-1)(x+1) \ge 0$

x		-4		-1		1		2	
$x+1$	$-$	‖	$-$	0	$+$	‖	$+$	‖	$+$
$x-1$	$-$	‖	$-$		$-$	0	$+$	‖	$+$
$x-2$	$-$	‖	$-$		$-$		$-$	0	$+$
Γιν.	$-$	‖	$-$		$+$	‖	$-$	‖	$+$

Άρα $x \in (-1, 1) \cup (2, +\infty)$

ΘΕΜΑ Δ

Δ1 $f(x)=g(x) \Leftrightarrow 2e^{2x}+1 = e^{x}e + e^{2x} + e^{x} - e + 1$

$\Leftrightarrow e^{2x} - e^{x}(e+1) + e = 0 \Leftrightarrow y^2 - (e+1)y + e = 0$ $\quad \Delta = (e+1)^2 - 4e = e^2+2e+1-4e = (e-1)^2$

$$y_{1,2} = \begin{cases} \dfrac{e+1+e-1}{2} = \dfrac{2e}{2} = e \rightarrow e^{x}=e \Leftrightarrow \boxed{x=1} \\ \dfrac{e+1-e+1}{2} = \dfrac{2}{2} = 1 \rightarrow e^{x}=1 \Leftrightarrow \boxed{x=0} \end{cases}$$

Δ2 $f(x)>1 \Leftrightarrow \dfrac{2e^{2x}+1}{e^{x}+2} > 1 \overset{e^x+2>0}{\Leftrightarrow} 2e^{2x}+1 > e^{x}+2 \Leftrightarrow 2e^{2x}-e^{x}-1>0$

$\Leftrightarrow 2y^2 - y - 1 > 0, \quad \Delta = 1+8 = 9, \quad y_{1,2} = \begin{cases} \dfrac{1+3}{4} = 1 \\ \dfrac{1-3}{4} = \dfrac{-2}{4} = -\dfrac{1}{2} \end{cases}$

	$-1/2$		1	
$+$	0	$-$	0	$+$

$y < -\frac{1}{2}$ ή $y>1 \Leftrightarrow e^{x}>1 \Leftrightarrow \boxed{x>0}$

$e^{x} < -\frac{1}{2}$

ΑΔΥΝΑΤΗ

Δ3 $f(0) = \dfrac{2+1}{1+2} = 1$: $\quad \alpha^{x} \leq 1 \begin{cases} \text{Αν } \alpha \in (0,1): \alpha^{x} < \alpha^{0} \Leftrightarrow x>0 \\ \text{Αν } \alpha \in (1,+\infty): \alpha^{x} < \alpha^{0} \Leftrightarrow x<0 \end{cases}$

Δ4 $συν(2023π - x) = συν(π - x) = -συνx$

α $\dfrac{2e^{2x}+1}{\dfrac{2e^{2x}+1}{e^{x}+2}} + e^{-x} - 2συνx = 2 \Leftrightarrow e^{x} + \cancel{2} + e^{-x} = 2συνx + \cancel{2}$

$\Leftrightarrow e^{x} + e^{-x} = 2συνx$

b $-2 \leq 2συνx \leq 2 \Leftrightarrow -2 \leq e^{x} + e^{-x} \leq 2 \Leftrightarrow e^{x} + \dfrac{1}{e^{x}} \leq 2 \overset{\cdot e^x>0}{\Leftrightarrow}$

$e^{2x}+1 \leq 2e^{x} \Leftrightarrow e^{2x} - 2e^{x} + 1 \leq 0 \Leftrightarrow (e^{x}-1)^2 \leq 0 \Leftrightarrow e^{x}-1=0 \Leftrightarrow \boxed{x=0}$

ΕΠΑΛΗΘΕΥΣΗ: $e^{0} + e^{0} = 2συν0 \Leftrightarrow 1+1=2$ ΙΣΧΥΕΙ

ΡΙΖΑ 2

άρα $\boxed{x=0}$