ΑΠΑΝΤΗΣΕΙΣ - ΔΙΑΓΩΝΙΣΜΑ ΑΛΓΕΒΡΑ Α' ΛΥΚΕΙΟΥ

12/10/25

ΘΕΜΑ Α

Α1. σελ. 15, Σχολικό Βιβλίο

Α2. (α) (i) $\varnothing' = \Omega$ (ii) $\Omega' = \varnothing$ (iii) $(A')' = A$

(β) (i) $A \cap B = A$ (ii) $A \cup B = B$

Α3. (α) Λ

(β) Λ

(γ) Λ

(δ) Σ

(ε) Σ

ΘΕΜΑ Β

Β1. (α) (i) $a^2 = 2a + b \ (1)$, $b^2 = 2b + a \ (2)$

↳ 1ο μέλος: $a^2 - b^2 \overset{(1)}{\underset{(2)}{=}} 2a + b - (2b + a) = 2a + b - 2b - a$

$\Leftrightarrow a^2 - b^2 = a - b$ (2ο μέλος)

(ii) Από (i) έχουμε: $a^2 - b^2 = a - b \Leftrightarrow (a-b)\cdot(a+b) - (a-b) = 0$

$\Leftrightarrow (a-b)\cdot(a+b-1) = 0 \underset{a-b\neq 0}{\overset{a\neq b}{\Longleftrightarrow}} a + b - 1 = 0 \Leftrightarrow \boxed{a + b = 1}$

(β) $A = a^2 + b^2 \overset{(1)}{\underset{(2)}{=}} 2a + b + 2b + a = 3a + 3b$

$\Leftrightarrow A = 3\cdot(a+b) \overset{(ii)}{=} 3 \cdot 1 \Leftrightarrow \boxed{A = 3}$

Β2. (α) $2a - 1$, $b - 1$: αντίστροφοι, οπότε βάσει ορισμού θα ισχύει: $(2a-1)\cdot(b-1) = 1 \Leftrightarrow 2ab - 2a - b + \cancel{1} = \cancel{1}$

$\Leftrightarrow 2ab = 2a + b$ ή $\boxed{2a + b = 2ab}$

(β) Αρκεί να αποδείξουμε ότι: $x + y = 0$

$\Leftrightarrow a - b + a\cdot(1 - 2b) + 2b = 0$

$\Leftrightarrow a - b + a - 2ab + 2b = 0 \Leftrightarrow 2a + b - 2ab = 0$

$\overset{(α)}{\Leftrightarrow} 2ab - 2ab = 0$

ΘΕΜΑ Γ

Γ1. (i) $\dfrac{2x-4}{x^2+4x+4} : \dfrac{x^2-4x+4}{x^2-4} = \dfrac{2\cdot(x-2)}{(x+2)^2} : \dfrac{(x-2)^2}{(x-2)(x+2)} = \dfrac{2\cdot(x-2)}{(x+2)^2}\cdot\dfrac{(x-2)(x+2)}{(x-2)^2}$

$= \dfrac{2\cdot\cancel{(x-2)}\cdot\cancel{(x-2)}\cancel{(x+2)}}{(x+2)^{\cancel{2}}\cdot(x-2)^{\cancel{2}}} = \boxed{\dfrac{2}{x+2}}$

(ii) $\left(\dfrac{a^2x-\beta^2}{x^2-1} + \dfrac{\beta^2x-a^2}{1-x^2}\right)\cdot\dfrac{x^2-2x+1}{\underbrace{ax+\beta x}-a-\beta}$

$= \left(\dfrac{a^2x-\beta^2}{x^2-1} - \dfrac{\beta^2x-a^2}{x^2-1}\right)\cdot\dfrac{(x-1)^2}{x\cdot(a+\beta)-(a+\beta)} = \dfrac{[a^2x-\beta^2-(\beta^2x-a^2)]}{x^2-1}\cdot\dfrac{(x-1)^{\cancel{2}}}{(a+\beta)\cancel{(x-1)}}$

$= \dfrac{a^2x-\beta^2-\beta^2x+a^2}{(x-1)(x+1)}\cdot\dfrac{x-1}{a+\beta} = \dfrac{x\cdot(a^2-\beta^2)+a^2-\beta^2}{(x-1)(x+1)}\cdot\dfrac{x-1}{a+\beta}$

$= \dfrac{(a^2-\beta^2)\cdot\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}\cdot\dfrac{x-1}{a+\beta} = \dfrac{(a-\beta)\cdot\cancel{(a+\beta)}\cdot\cancel{(x-1)}}{\cancel{(x-1)}\cdot\cancel{(a+\beta)}} = \boxed{a-\beta}$

Γ2. (i) 1ο μέλος: $(a+1)^2-(a-1)(a+1) = a^2+2a+1-(a^2-1) =$

$a^2+2a+1-\cancel{a^2}+1 = 2a+2 = 2\cdot(a+1)$ (2ο μέλος)

(ii) 1ο μέλος: $(\beta+2)^3-6(\beta+1)^2 = \beta^3+3\beta^2\cdot2+3\cdot\beta\cdot2^2+2^3-6\cdot(\beta^2+2\beta+1)$

$= \beta^3+\cancel{6\beta^2}+\cancel{12\beta}+8-\cancel{6\beta^2}-\cancel{12\beta}-6 = \beta^3+2$ (2ο μέλος)

Γ3. (i) $A = \dfrac{a^3-\beta^3}{(a+\beta)^2-a\beta} = \dfrac{(a-\beta)(a^2+a\beta+\beta^2)}{a^2+2a\beta+\beta^2-a\beta} = \dfrac{(a-\beta)\cdot\cancel{(a^2+a\beta+\beta^2)}}{\cancel{a^2+a\beta+\beta^2}}$

$\Leftrightarrow \boxed{A=a-\beta}$

(ii) Από (i) όπου $a=99$ και $\beta=19$ έχουμε:

$A = \dfrac{99^3-19^3}{(99+19)^2-99\cdot19} = \dfrac{99^3-19^3}{118^2-99\cdot19} = 99-19=80$

Άρα $\boxed{B=80}$

ΘΕΜΑ Δ

Δ1. Εφαρμογή 14, σελ. 19 βιβλίο Ε.Δ.

Δ2.

$$A = (16\alpha+32\beta)^{-2} - (32\alpha+64\beta)^{-2} + \left[\left(-\frac{2}{3}\right)^{-4} : 3^4\right]^2$$

$$A = [16(\alpha+2\beta)]^{-2} - [32(\alpha+2\beta)]^{-2} + \left[\left(-\frac{3}{2}\right)^{4} : 3^4\right]^2$$

$$A = \left(16\cdot\frac{1}{2}\right)^{-2} - \left(32\cdot\frac{1}{2}\right)^{-2} + \left(\frac{\cancel{3^4}}{2^4}\cdot\frac{1}{\cancel{3^4}}\right)^2$$

$$A = 8^{-2} - 16^{-2} + (2^{-4})^2 \Leftrightarrow A = (2^3)^{-2} - (2^4)^{-2} + 2^{-8}$$

$$A = 2^{-6} - \cancel{2^{-8}} + \cancel{2^{-8}} \Leftrightarrow A = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$\Leftrightarrow \boxed{A = \frac{1}{64}}$$

Δ3.

i) (1) $(-1)^{2024}\cdot(\alpha+\beta)^2 + (-1)^{2025}\cdot(\alpha-\beta)^2 = 4$

$$\Leftrightarrow (\alpha+\beta)^2 - (\alpha-\beta)^2 = 4 \Leftrightarrow \alpha^2+2\alpha\beta+\beta^2 - (\alpha^2-2\alpha\beta+\beta^2) = 4$$

$$\Leftrightarrow \cancel{\alpha^2} + 2\alpha\beta + \cancel{\beta^2} - \cancel{\alpha^2} + 2\alpha\beta - \cancel{\beta^2} = 4 \Leftrightarrow 4\alpha\beta = 4 \Leftrightarrow \boxed{\alpha\cdot\beta = 1}$$

Άρα α, β αντίστροφοι

ii) (2) $(\alpha-\beta-1)\cdot(\alpha+\beta-1) = -2(1+\alpha-\alpha^2)$

$$\Leftrightarrow (\alpha-1-\beta)\cdot(\alpha-1+\beta) = -2 - 2\alpha + 2\alpha^2$$

$$\Leftrightarrow (\alpha-1)^2 - \beta^2 = -2 - 2\alpha + 2\alpha^2 \Leftrightarrow \alpha^2 - \cancel{2\alpha} + 1 - \beta^2 = -2 - \cancel{2\alpha} + 2\alpha^2$$

$$\Leftrightarrow \boxed{\alpha^2+\beta^2 = 3}$$

Οπότε: $A = (\alpha+\beta)^2 = \underbrace{\alpha^2 + 2\overset{1}{\alpha\beta} + \beta^2}_{3} = 3 + 2\cdot 1 = \boxed{5}$

iii) $B = \alpha^4 + \beta^4 = (\alpha^2)^2 + (\beta^2)^2 \overset{(*)}{=} (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$

$$\Leftrightarrow B = 3^2 - 2\cdot 1^2 = 9 - 2 \Leftrightarrow \boxed{B = 7}$$

(*) Αποδεικνύεται ότι: $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$