

ΛΥΣΕΙΣ ΔΙΑΓΩΝΙΣΜΑΤΟΣ ΑΛΓΕΒΡΑΣ

Α' ΛΥΚΕΙΟΥ 11/10/26

ΘΕΜΑ 1

A1) ΣΧΟΛΙΚΟ ΣΕΛ. 17

A2) α) $(A')' = A$ β) $(\emptyset)' = \phi$ γ) $\emptyset \cup \phi = \emptyset$ δ) $(\phi)' = \emptyset$
ε) $\emptyset \cap \phi = \phi$

A3) α) $(-a-b)^3 = -(a+b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3$

β) $(a+b)(b-a) = b^2 - a^2$

γ) $(a+b+x)^2 = a^2 + b^2 + x^2 + 2ab + 2bx + 2xa$

δ) $-1 - 1 + 1 = -1$

ε) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

A4) α) 1 β) 1 γ) I δ) I ε) 1

ΘΕΜΑ 2

B1) $a^2 = 2a + b$ ή $b^2 = 2b + a$, $a \neq b$

α) i) $a^2 - b^2 = 2a + b - 2b - a = a - b$

ii) $a^2 - b^2 = a - b \Leftrightarrow (a-b)(a+b) = a-b \xrightarrow{a-b \neq 0} a+b = 1$

β) $A = a^2 + b^2 = 2a + b + 2b + a = 3a + 3b = 3(a+b) = 3 \cdot 1 = 3$

B2) α) $(x-2y)^2 - 2(3-2xy) = 5y^2 - 1 \Leftrightarrow x^2 - 4xy + 4y^2 - 6 + 4xy = 5y^2 - 1 \Leftrightarrow x^2 - y^2 = 5$

β) $P = (x+y)^3(x-y)^3 = ((x+y)(x-y))^3 = (x^2 - y^2)^3 = 5^3 = 125$

ΘΕΜΑ 3

$$\Gamma_1) \alpha) A = \frac{x^4 + x^3 - 4x^2 - 4x}{(x^3 - x)(x - 2)} = \frac{x^3(x+1) - 4x(x+1)}{x(x^2 - 1)(x - 2)} =$$

$$\frac{x(x+1)(x^2 - 4)}{x(x-1)(x+1)(x-2)} = \frac{x(x+1)(x-2)(x+2)}{x(x-1)(x+1)(x-2)} = \frac{x+2}{x-1}$$

$$\beta) B = \frac{x^2 - 9}{x^2 - 6x + 9} : \left(\frac{x-3}{x+3} \right)^{-3} = \frac{(x-3)(x+3)}{(x-3)^2} \cdot \left(\frac{x+3}{x-3} \right)^{-3} =$$

$$\frac{x+3}{x-3} \cdot \frac{(x-3)^3}{(x+3)^3} = \frac{(x-3)^2}{(x+3)^2}$$

$$\Gamma_2) \Gamma = \frac{x^2 y^{-2} x^{-4} y^8 x y^{-2}}{x^{-1} y x^6 y^{-3} x^{-2} y} : (x^{-2} y^9) =$$

$$\frac{x^{-1} y^4}{x^3 y^{-1}} : x^{-2} y^9 = \frac{x^{-4} y^5}{x^{-2} y^9} = x^{-2} y^{-4} = (xy^2)^{-2}$$

$$\text{οπδ } \Gamma = (-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

$$\Gamma_3) A' = \{4\}$$

$$A' \cup B = \{3, 4\}$$

$$(A \cap B) = 3 \quad \text{οπδ} \quad (A \cap B)' = \{1, 2, 4\}$$

ΘΕΜΑ 4

$$\Delta 1) \quad 2(a+b)^2 + (a-b)^2 - (a+b)(a-b) = a^2 + 3b^2 (=1)$$

$$2(a^2 + 2ab + b^2) + a^2 - 2ab + b^2 - (a^2 - b^2) = a^2 + 3b^2 (=1)$$

$$\underline{2a^2} + \underline{4ab} + \underline{2b^2} + \underline{a^2} - \underline{2ab} + \underline{b^2} - \underline{a^2} + \underline{b^2} = \underline{a^2} + \underline{3b^2} (=1)$$

$$a^2 + 2ab + b^2 = 1 \quad (a+b)^2 = 0 \quad (=) \quad a+b=0 \quad \text{όρα}$$

a, b αντίθετοι

$$\Delta 2) \quad \text{α)} \quad A = \frac{a^3 - 4ab^2}{a^3 - a^2b - 2ab^2} = \frac{a(a^2 - 4b^2)}{a(a^2 - ab - 2b^2)} =$$

$$\frac{(a-2b)(a+2b)}{a^2 - ab - b^2 - b^2} = \frac{(a-2b)(a+2b)}{(a-b)(a+b) - b(a+b)} =$$

$$\frac{(a-2b)(a+2b)}{(a+b)(a-b-b)} = \frac{(a-2b)(a+2b)}{(a+b)(a-2b)} = \frac{a+2b}{a+b}$$

$$\text{β)} \quad A = \frac{a+2b}{a+b} \quad \underline{a=3b} \quad \frac{3b+2b}{3b+b} = \frac{5b}{4b} = \frac{5}{4}$$

$$\Delta 3) \quad \frac{a^2+b}{b^2+a} = \frac{a}{b} \quad (=) \quad a^2b + b^2 = ab^2 + a^2 (=)$$

$$\underline{a^2b} + \underline{b^2} - \underline{ab^2} - \underline{a^2} = 0 \quad (=) \quad ab(a-b) + (b-a)(b+a) = 0 (=)$$

$$ab(a-b) - (a-b)(b+a) = 0 \quad (=) \quad (a-b)(ab - b - a) = 0$$

$$\underline{a \neq b} \quad ab - b - a = 0 \quad (=) \quad b + a = ab \quad (=) \quad \underline{ab}$$

$a-b \neq 0$

$$\frac{b}{ab} + \frac{a}{ab} = \frac{ab}{ab} \quad (=) \quad \frac{1}{a} + \frac{1}{b} = 1$$